Trade and the End of Antiquity

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What we do

1. We use ancient coins to learn about Late Antiquity economics, 4th-10th century AD
2. Structural mapping from coins to economics: dynamic model of trade and money
3. For each 20-year period, 13 regions, structural estimates for:
    a. bilateral trade flows
    b. technology, geography, and trade imbalances
    c. real consumption
4. We validate our estimates with (scant) known historical facts:
    a. urbanization in Europe post 700 AD
    b. declining trade between Islam and Christianity
    c. Byzantine monetary (and economic) collapse
    d. Relative rise of Islamic Spain, Carolingian northern Europe, Eastern Caliphate

Outline

Data

Model

Structural estimation

Results

Coin dataset

- Data: (2/3 from FLAME (2023), 1/3 manually from catalogues)
  - 270,500 coins
  - in 5,609 hoards
  - buried between AD 325 and AD 950
  - in the extended Mediterranean
- Relevant information:
  - Hoard location: h
  - For each coin, mint location ("birthplace"): m
  - For each coin, mint date ("birthdate"): t
  - Youngest coin in the hoard, *terminus post quem (tpq)*, $T = \sup t$

Sources Example

Spatial distribution

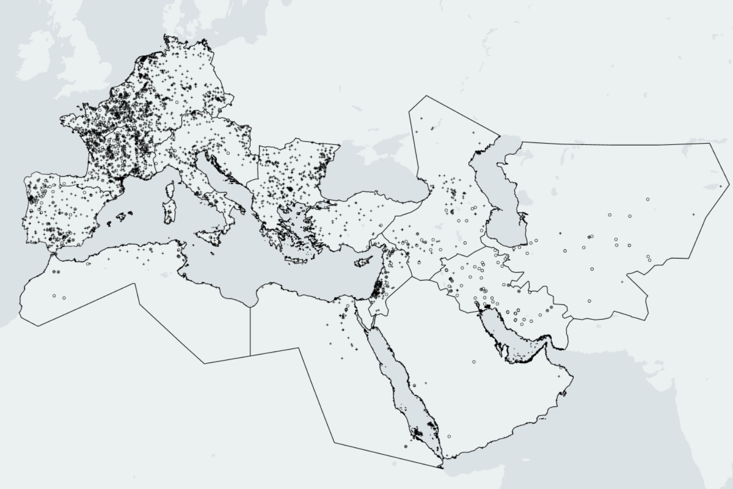

Figure: Region definitions, mints (○), and hoards (+)

▸ Temporal distribution

Coin flows before the Arab conquests

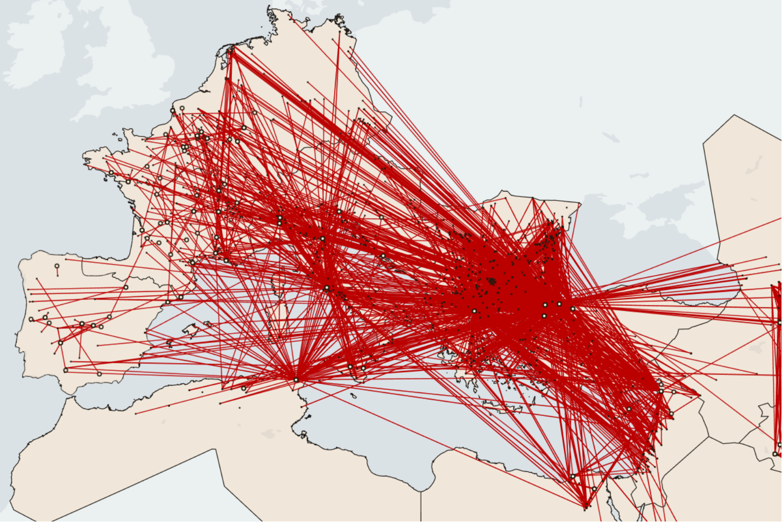

Figure: Before the Arab conquests: AD 450-630

▸ Gravity ▸ Age-distance

Coin flows after the Arab conquests

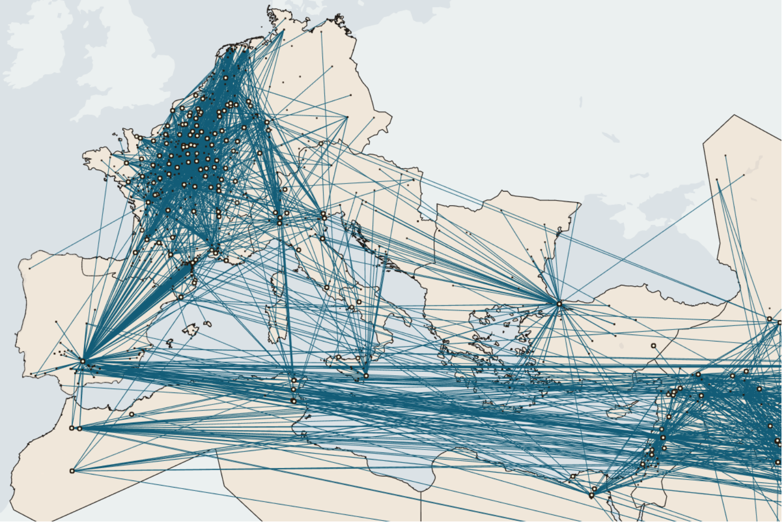

Figure: After the Arab conquests: AD 713-900

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Inter- and intra-temporal equilibrium (simplified case: no saving)

- Goods market clearing, denominated in coins

$$\overbrace{w_i L_i[t]}^{income_i[t]} = \sum_n \pi_{ni}[t] \overbrace{\Big((1-\lambda)\, w_n L_n[t-1] + M_n[t]\Big)}^{expenditure_n[t]}, \forall i, t \tag{1}$$

$$\Leftrightarrow S_i[t] = \sum_n \pi_{ni}[t] \Big((1-\lambda)\, S_n[t-1] + M_n[t]\Big), \forall i, t \tag{2}$$

w_i: wages; L_i: labor; π_{ni}: expenditure shares; λ: coin loss; M_n: minting; S_i: coin stocks

- Fraction π_{ni} of n's expenditure allocated to goods from i (Eaton and Kortum, 2002)

$$\pi_{ni}[t] = \frac{\left(T_i[t]\, w_i^{-\theta}[t]\right)\left(d_{ni}[t]\right)^{-\theta}}{\sum_k \left(T_k[t]\, w_k^{-\theta}[t]\right)\left(d_{nk}[t]\right)^{-\theta}}, \forall i, n, t \tag{3}$$

T_i: technology; d_{ni}: trade cost; θ: trade elasticity

Assumptions: (1) monetized economy + (2) coins are fungible + (3) EK trade

stationary equilibrium with or w/o saving

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T_i: technology; d_{ni}: trade cost; θ: trade elasticity
Assumptions: (1) monetized economy + (2) coins are fungible + (3) EK trade

▸ stationary equilibrium with or w/o saving

Composition of coin stocks (prediction A)

- Stock $S_i[T]$ composed of different coin types, $S_i[T] = \sum_{m=1}^{N} \sum_{t \leq T} S_{mi}[t, T]$
- Coins start their 'coin life' when they are minted, $S_{mm}[t, t] = M_m[t]$
- Then stocks evolves recursively,

$$S_{mi}[t, \tau] = (1 - \lambda) \sum_{n=1}^{N} \pi_{ni}[\tau] \, S_{mn}[t, \tau - 1] \tag{4}$$

- Recursive solution in matrix form (coin origin $\times$ coin destination),

$$\mathbf{S}[t, T] = (1 - \lambda) \, \boldsymbol{M}[t] \, \boldsymbol{\Pi}[t+1] \, \boldsymbol{\Pi}[t+2] \cdots \boldsymbol{\Pi}[T] \tag{4'}$$

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Data | Leontief inverse | trade/coin flows with saving

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- Recursive solution in matrix form (coin origin $\times$ coin destination),

$$\mathbf{S}[t, t+2] = (1 - \lambda)^2 \boldsymbol{M}[t] \, \boldsymbol{\Pi}[t+1] \, \boldsymbol{\Pi}[t+2] \cdots \boldsymbol{\Pi}[T] \tag{4'}$$

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$$\mathbf{S}[t, T] = (1 - \lambda)^{T-t} \boldsymbol{M}[t] \, \boldsymbol{\Pi}[t+1] \, \boldsymbol{\Pi}[t+2] \cdots \boldsymbol{\Pi}[T] \tag{4'}$$

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Mapping from model to data

- *Time*: 20-year time intervals from AD 320 to AD 920
- *Space*: $N = 13$ regions around the Mediterranean ▸ map
- *Coin exit rate*: $\lambda = 0.301$ (1.7% per year) from exponential decay of coins ▸ age distribution
- *Data generating process*: hoards ($\boldsymbol{H}$) are random samples from coin stocks ($\boldsymbol{S}$)

$$\mathbb{E}\left[\frac{H_{m,h}[t,T]}{H_h[T]}\right] = \frac{S_{m,h}[t,T]}{S_h[T]}$$

Trade cost function

- *Assumption* (4) Parameterize trade cost: $d_{nn}[t] = 1, \forall n, t$ and,

$$\ln\left((d_{ni}[t])^{-\theta}\right) = \gamma_0 + \zeta \ln(TravelTime_{ni}) + \kappa_1 PoliticalBorder_{ni}[t] + \kappa_2 ReligiousBorder_{ni}[t]$$

- *Travel times*: optimal routing with ancient technology (time-invariant) map
- *Political borders*: going across 19 polities (time-varying)
- *Religious border*: crossing the Islamic border (time-varying) map

with saving

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with saving

Maximum likelihood estimation

- If hoards are random samples from stocks, then multinomial distribution within each hoard

$$\Pr\left(\cdots, H_{mh}[t,T], \cdots\right) = \frac{H_h[T]!}{\prod_{m,t\leq T} H_{mh}[t,T]!} \prod_{m,t\leq T} \left(\frac{S_{mh}[t,T]}{\sum_{m',t'\leq T} S_{m'h}[t',T]}\right)^{H_{mh}[t,T]}, \forall h, T$$

Clever trick: coin shares within a hoard → purges endogenous selection of hoards.

- Parameters to be estimated (minting, seller terms, trade cost parameters),

$$\Theta = \left(\left(\cdots, M_n[t], \cdots\right)_{n\neq n_0, t\neq t_0}, \left(\cdots, T_n w_n^{-\theta}[t], \cdots\right)_{n\neq n_0, t}, \gamma_0, \zeta, \kappa_1, \kappa_2\right).$$

Normalizations: $M_{n_0}[t_0] = 100$ (Northern Italy, 320-40), and $T_{n_0} w_{n_0}^{-\theta}[t] = 100, \forall t$.

- Maximize log-likelihood of observing hoard dataset $\boldsymbol{H}$ given parameters $\boldsymbol{\Theta}$,

$$\widehat{\Theta} = \arg\max_{\Theta} \sum_{h,T} \sum_{m,t\leq T} H_{mh}[t,T] \left(\ln S_{mh}[t,T](\Theta) - \ln \sum_{m',t'\leq T} S_{m'h}[t',T](\Theta)\right)$$

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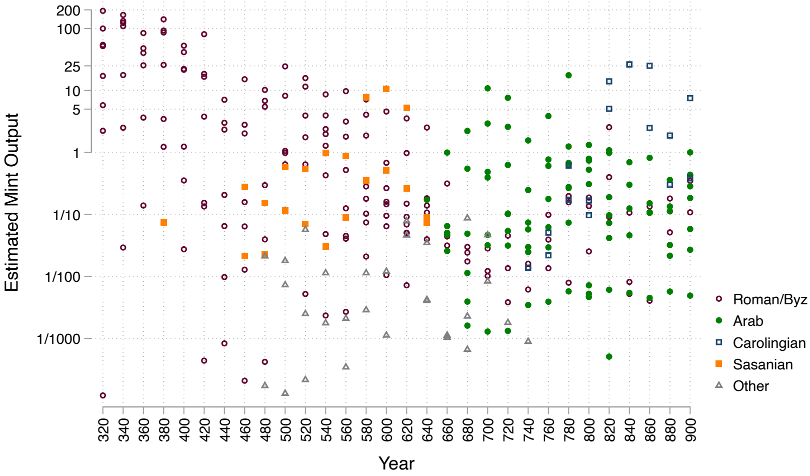
Estimation results: Determinants of trade costs

$$\ln\left((d_{ni}[t])^{-\theta}\right) = constant - \underset{(0.02)}{3.03}\ln(TravelTime_{ni}) - \underset{(0.02)}{0.49}\,PoliticalBorder_{ni}[t]$$

$$- \underset{(0.12)}{1.97}\,ReligiousBorder^{East}_{ni}[t] - \underset{(0.22)}{4.59}\,ReligiousBorder^{West}_{ni}[t] - \underset{(0.18)}{5.2}\,ReligiousBorder^{Med.}_{ni}[t]$$

- Travel time elasticity similar to estimates on ancient trade $\in (1.91, 2.89)$.
  Roman trade: Flückiger et al. (2022); Bronze Age trade: Barjamovic et al. (2019).
- Political border tax: 13% (with $\theta = 4$)
- Religious border taxes: East 64% | West 215% | Mediterranean 267% (with $\theta = 4$)
- Anderson and van Wincoop (2003) US-Canada border tax: 49% (with $\theta = 4$)

Estimation results: Mint output



Normalization: $M_{n_0}[t_0] = 100$ (Northern Italy, AD 320-40).

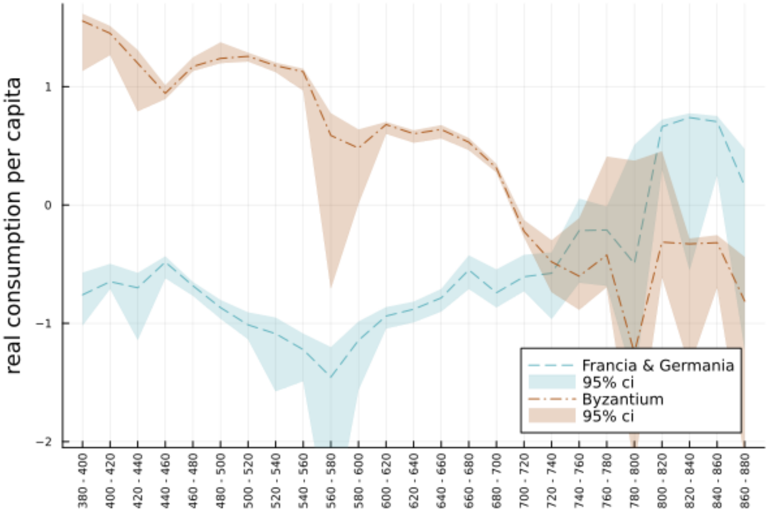
Real consumption per capita (prediction B)

$$\underbrace{\frac{X_n/p_n}{L_n}}_{\text{Real Consumption}} = \gamma^{-1} \underbrace{(\pi_{nn})^{-1/\theta}}_{\text{Openness}} \underbrace{(T_n)^{1/\theta}}_{\text{Technology}} \underbrace{\left(1 + \frac{M_n - \lambda w_n L_n}{w_n L_n}\right)}_{\text{Trade Deficit}} \qquad \text{(eq. 20)}$$

- **Assumption:** trade elasticity ($\theta = 4$) to derive aggregate real consumption (X_n/p_n)
- **Assumption:** "Malthusian force" ($L_n = T_n$) to separate L_n and $T_n^{1/\theta}$ from $L_n T_n^{1/\theta}$
- **Normalization:** units for $T_n^{1/\theta}$ chosen such that $\mathbb{E}_t\left[\frac{X_n[t]/p_n[t]}{L_n[t]}\right] \equiv 1, \forall t$

technical steps

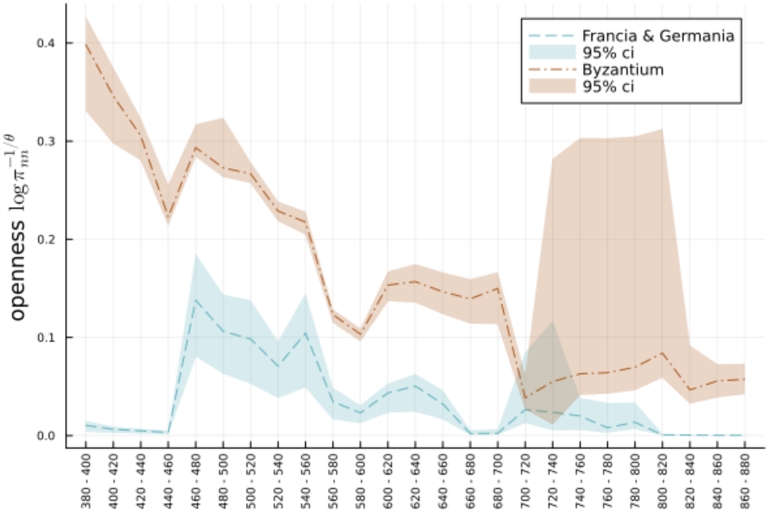
Byzantium vs northern Europe (380-880): *log real consumption*



Bootstrapped 95% confidence intervals. *Normalizations:* $\ln\left(\mathbb{E}_t\left[\frac{X_n[t]/p_n[t]}{L_n[t]}\right]\right) \equiv 0, \forall t.$

other regions

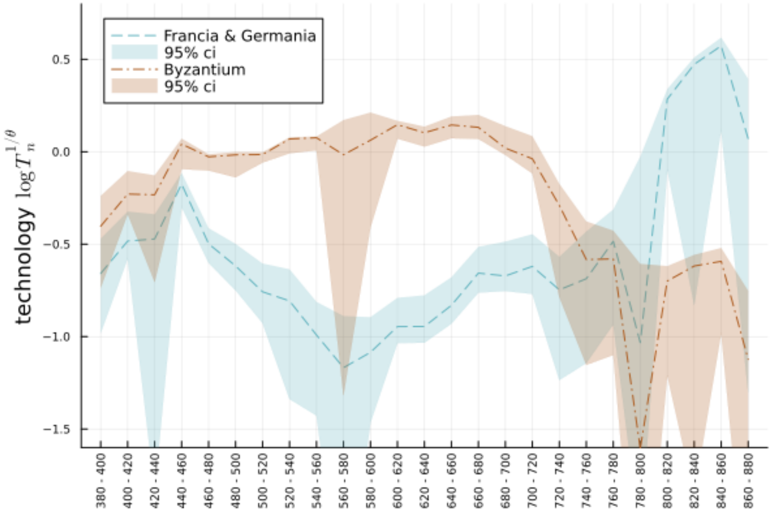
Byzantium vs northern Europe (380-880): *trade openness*



Bootstrapped 95% confidence intervals.

other regions

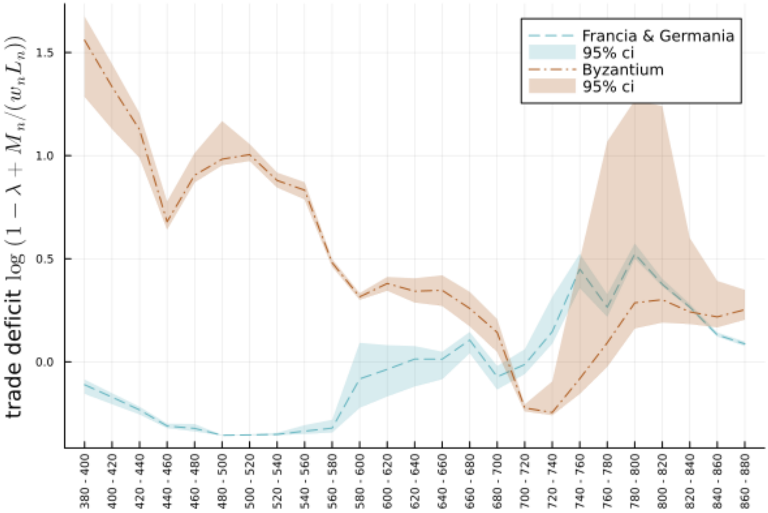
Byzantium vs northern Europe (380-880): *technology*



Bootstrapped 95% confidence intervals. *Normalizations:* $\ln\left(\mathbb{E}_t\left[\frac{X_n[t]/p_n[t]}{L_n[t]}\right]\right) \equiv 0, \forall t.$

other regions

Byzantium vs northern Europe (380-880): *trade deficits*



Bootstrapped 95% confidence intervals.

other regions

Real consumption: technology, geography, and trade (deficits)

Steady state equilibria: AD 460-620 and AD 700-900

	Consumption $\frac{X_n/p_n}{L_n}$		Import share $1-\pi_{nn}$		Technology $T_n^{1/\theta}$		Trade deficits $\frac{X_n - w_n L_n}{w_n L_n} = \frac{M_n}{w_n L_n} - \lambda$	
	460-620	700-900	460-620	700-900	460-620	700-900	460-620	700-900
Francia and Germania	**0.22**	**1.58**	**0.24**	**0.00**	**0.23**	**1.62**	**-0.14**	**-0.03**
	(0.02)	(0.16)	(0.05)	(0.00)	(0.03)	(0.17)	(0.04)	(0.01)
Byzantine Heartlands	**3.21**	**0.67**	**0.59**	**0.21**	**1.07**	**0.51**	**1.40**	**0.24**
	(0.11)	(0.20)	(0.02)	(0.13)	(0.04)	(0.06)	(0.12)	(0.47)
Arabian Peninsula	**0.22**	**0.69**	**0.07**	**0.00**	**0.31**	**0.84**	**-0.30**	**-0.18**
	(0.10)	(0.10)	(0.02)	(0.12)	(0.14)	(0.18)	(0.00)	(0.34)
Average *(all regions)*	**1.00**	**1.00**	**0.20**	**0.16**	**0.87**	**0.90**	**0.03**	**0.09**
	(0.00)	(0.00)	(0.01)	(0.02)	(0.01)	(0.06)	(0.01)	(0.14)

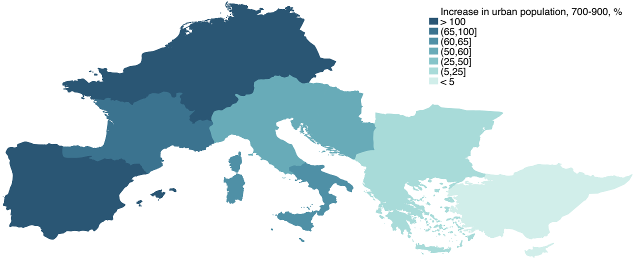
Normalizations: $\mathbb{E}_t\left[\frac{X_n[t]/p_n[t]}{L_n[t]}\right] \equiv 1, \forall t$. Bootstrapped s.e.'s in parentheses (100 bootstraps).

changes and counterfactuals

Real consumption changes vs European urbanization (AD 700–900)

Top:
$\Delta \log \left(\frac{X_n/p_n}{L_n} \right)$
from pre- to post-AD 700

Bottom:
Urban population change
post-AD 700
(urban: $>$ 1k inhabitants)
data from Buringh (2021)



THANK YOU!

BACKUP SLIDES

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Coins around the Mediterranean, AD 325 to AD 950

1. FLAME (2023) project:
   - "Framing the Late Antique and Early Medieval Economy."
   - Collaboration of $>$60 historians and numismatists.
   - $\sim$200,000 coins with complete records from $\sim$4,600 hoards
   - AD 325-725.
2. Hand-coded records from coin catalogs:
   - 797 coin finds.
   - 100,478 coins.
   - AD 725-950.

Data covers most of published literature on hoard records (and more)

Back

Coin hoard data: an example from al 'Ush (1972)

No.	MINT	DATE	DIAM.	WEIGHT	NUMB.
51	الأندلس	114	29.	2.93	4
52	"	115	29.5	2.92	1
53	"	116	26.5	2.92	3

Excerpt of an original publication on the Damascus silver hoard *:

- record number (51)
- mint (al-Andalus) *
- mint date (year 114 of the Hijri calendar) *
- diameter (29mm)
- weight (2.93g)
- number of coins with these attributes (4)

The issuing dynasty (Umayyad) is given in the table headings and the denomination and material (silver *dirham*) is stated in the text.
* required: mint (birth) place and date, hoard (death) place and date (*tpq*)

Back

Temporal distribution: coin hoards by burial dates (tpq)

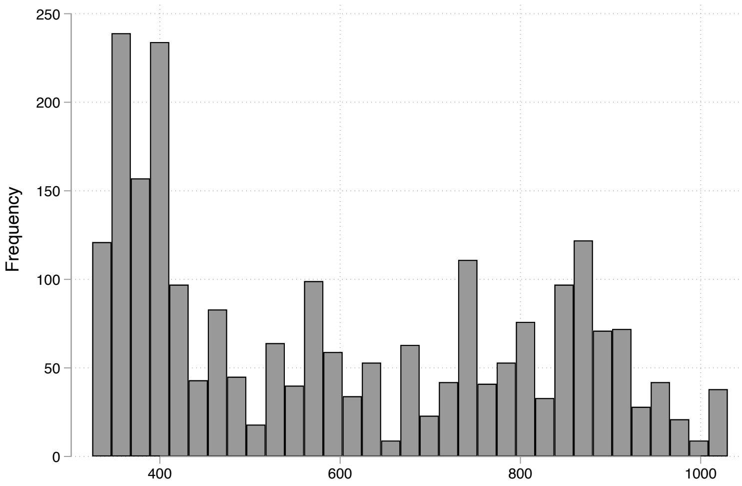

Figure: Terminus Post Quem (*tpq*) of hoards

Back

Gravity structure of coin flows (looks like trade)

	Dep. var.: # $\text{Coins}_{mhp(m)}$				Dep. var.: $\text{Value}_{mhp(m)}$	
	(1)	(2)	(3)	(4)	(5)	(6)
Log Distance	-1.138***	-1.002***	-1.139***	-0.954***	-1.146***	-0.991***
	(0.12)	(0.13)	(0.10)	(0.077)	(0.076)	(0.069)
Political border		-1.945***		-2.071***		-1.516***
		(0.62)		(0.47)		(0.27)
Hoard Cell FE	Yes	Yes	Yes	Yes	Yes	Yes
Mint $\times$ Empire Cell FE	Yes	Yes	Yes	Yes	Yes	Yes
Sample	All	All	Gold only	Gold only	Gold and Silver	Gold and Silver
Pseudo-R^2	0.767	0.778	0.809	0.824	0.800	0.810
Observations	217748	217748	57457	57457	146766	146766

Standard errors in parentheses, clustered at mint cell $\times$ empire and hoard cell level.

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Estimating equation (PPML):

$$\text{count}_{mhp(m)} = \exp\left(a_1 \log \text{Distance}_{mh} + a_2 \text{AcrossBorder}_{mhp(m)} + b_{m\times p(m)} + b_h + u_{mhp(m)}\right)$$

back

Older coins travel further (suggests dynamic diffusion)

Table: Within-hoard distance travelled and age of coin at deposit

	Dependent variable: Log Distance between Mint and Hoard		
	(1)	(2)	(3)
Log Age of Coin	0.146***	0.0831***	0.0750**
	(0.044)	(0.026)	(0.030)
Hoard FE	Yes	Yes	Yes
Mint × 50-year-interval FE		Yes	
Mint × 25-year-interval FE			Yes
R^2	0.762	0.863	0.869
Observations	287257	287040	286884

Standard errors in parentheses, clustered at the hoard level.

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

back

Fact #1: Arab conquest disrupts Mediterranean trade

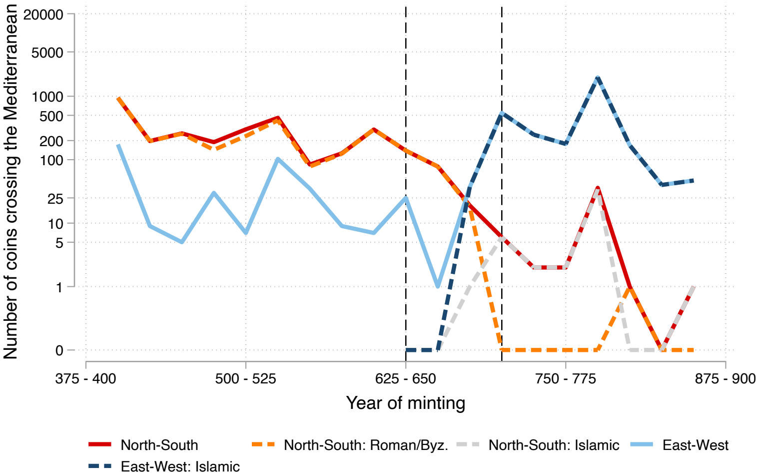

Figure: Number of coins flowing across the Mediterranean

Back

Arab conquest disrupts Mediterranean trade

Table: The Mediterranean Before and After the Arab Conquest

	Dependent variable: Number of Coins			
	(1)	(2)	(3)	(4)
Crossing Mediterranean × After Conquests	-1.893***	-3.246***	-0.662	-1.736
	(0.48)	(0.53)	(0.63)	(1.27)
Crossing Mediterranean × After Conquests × Islamic Coin		7.267***	4.789***	7.545***
		(0.90)	(0.95)	(0.89)
Crossing Mediterranean × After Conquests × Roman Coin			-3.287***	-2.893***
			(0.75)	(0.61)
Mint Cell × Empire FE	Yes	Yes	Yes	Yes
Mint Cell × Hoard Cell FE	Yes	Yes	Yes	Yes
After Conquests FE	Yes	Yes	Yes	
Mint Cell × After Conquests FE				Yes
Hoard Cell × After Conquests FE				Yes
Estimator	PPML	PPML	PPML	PPML
Observations	10480	10480	10480	6208

Standard errors in parentheses, clustered at the hoard × era and mint × era level.
$^{+}$ $p < 0.10$, * $p < 0.05$, ** $p < 0.01$

Estimating equation:

$$\text{count}_{mdht} = e^{(\gamma_1 \text{mediterranean}_{mh} \times \text{after}_t + \cdots + \alpha_{md} + \alpha_{mh} + \varepsilon_{mhdt})}$$

Back

Stationary steady state (without saving)

- Wages and trade shares jointly determined ($supply_i = demand_i$),

$$w_i L_i = \sum_n \pi_{ni} \Big(w_n L_n + M_n - \lambda w_n L_n \Big) \frac{1 - s_n}{1 - s_n + \lambda s_n}$$

$$\pi_{ni} = \frac{T_i \left(w_i d_{ni} \right)^{-\theta}}{\sum_k T_k \left(w_k d_{nk} \right)^{-\theta}}$$

- Constant aggregate stock of coins in circulation,

$$\sum_n M_n = \sum_n \lambda w_n L_n$$

- *Note*: trade deficits as in Dekle, Eaton and Kortum (2007),

$$D_n = X_n - w_n L_n = M_n - \lambda w_n L_n$$

Assumptions: (1) monetized economy + (2) coins are fungible + (3) EK trade

Back

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Assumptions: (1) monetized economy + (2) coins are fungible + (3) EK trade

Back

Distance elasticity by coin age

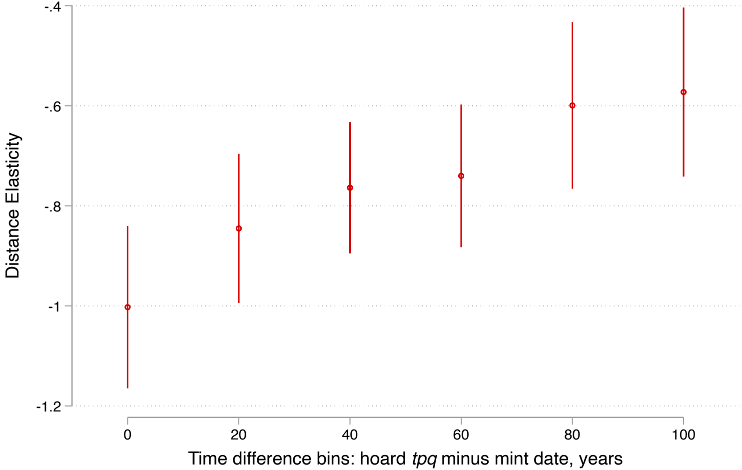


Figure: The distance elasticity declines as coins get older

Back

Coins vs trade (simpler steady state math)

- In a steady state, only age (a) matters,

$$\mathbf{S}\left[t,t+a\right]=\mathbf{S}\left[a\right]=\boldsymbol{M}\Big(\left(1-\lambda\right)\boldsymbol{\Pi}\Big)^{a},\forall t$$

- Combining coins (within a hoard) by origin-destination ignoring age,

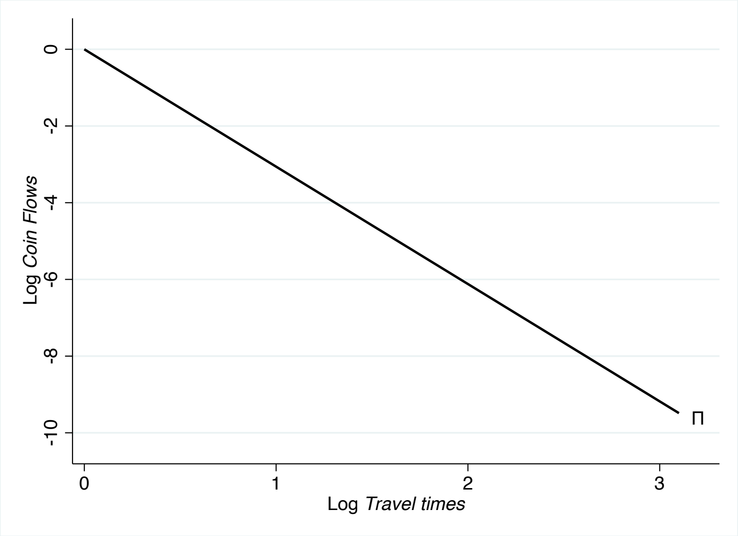
$$\mathbf{S}=\sum_{a=0}^{A}\mathbf{S}\left[a\right]=\boldsymbol{M}\left(\sum_{a=0}^{A}\Big(\left(1-\lambda\right)\boldsymbol{\Pi}\Big)^{a}\right)\underset{A\to+\infty}{=}\boldsymbol{M}\Big(\boldsymbol{I}-\left(1-\lambda\right)\boldsymbol{\Pi}\Big)^{-1}$$

- Naively ignoring age: Leontief inverse of trade flow matrix!

Assumptions: (1) monetized economy + (2) coins are fungible

back

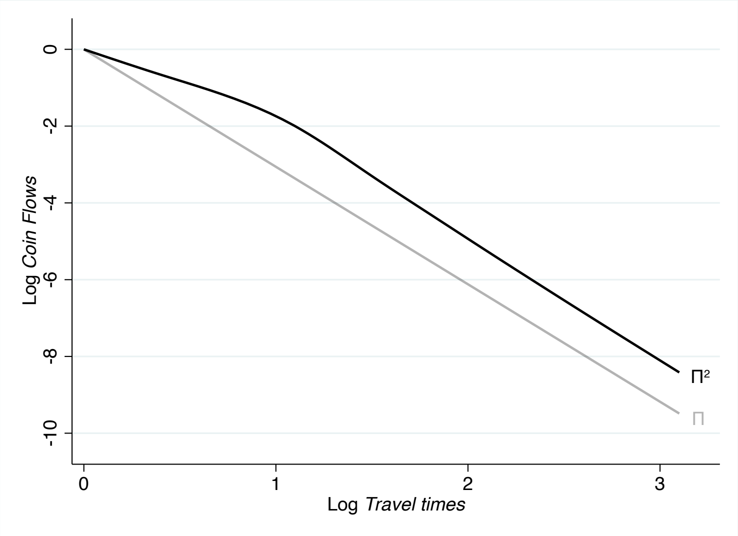
Coins vs trade (numerical example) ▸ Data



- Flow of coins: age 1 (same as trade flows $\mathbf{\Pi}$)

▸ back

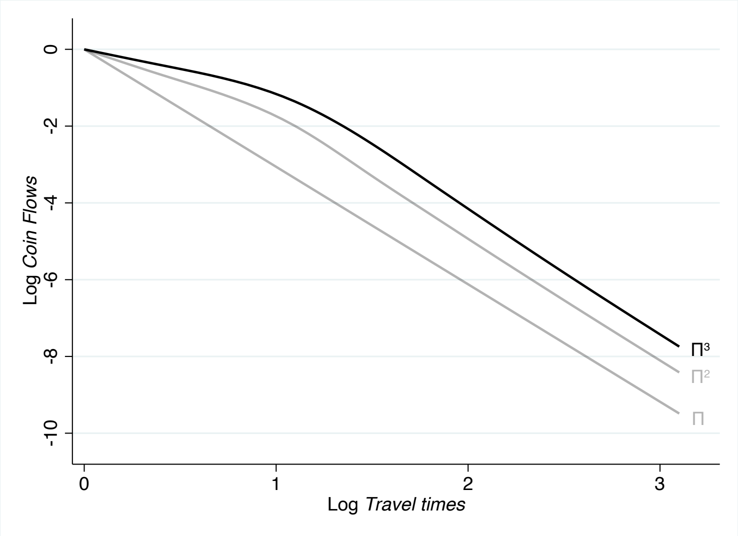
Coins vs trade (numerical example) ▸ Data



- Flow of coins: age 1, age 2

▸ back

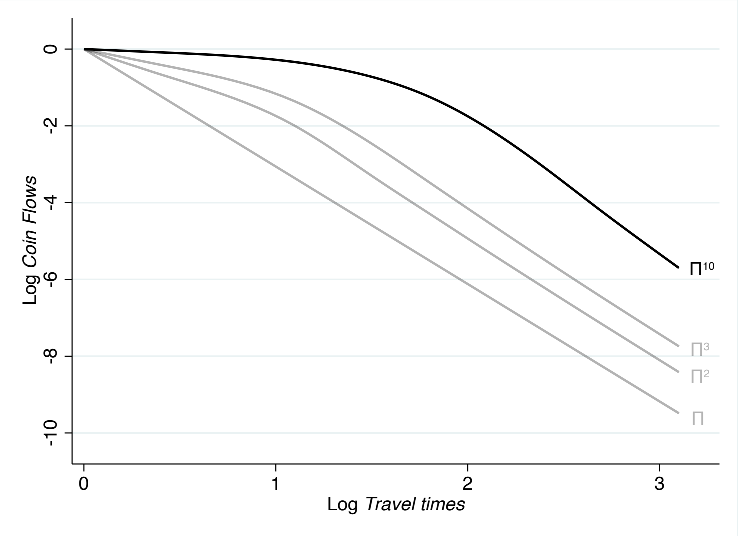
Coins vs trade (numerical example) ▸ Data



- Flow of coins: age 1, age 2, age 3

▸ back

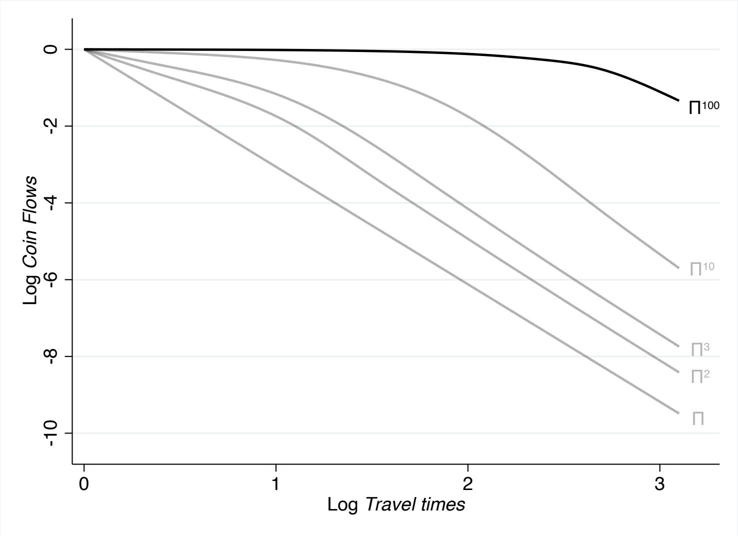
Coins vs trade (numerical example) ▸ Data



- Flow of coins: age 1, age 2, age 3, age 10

▸ back

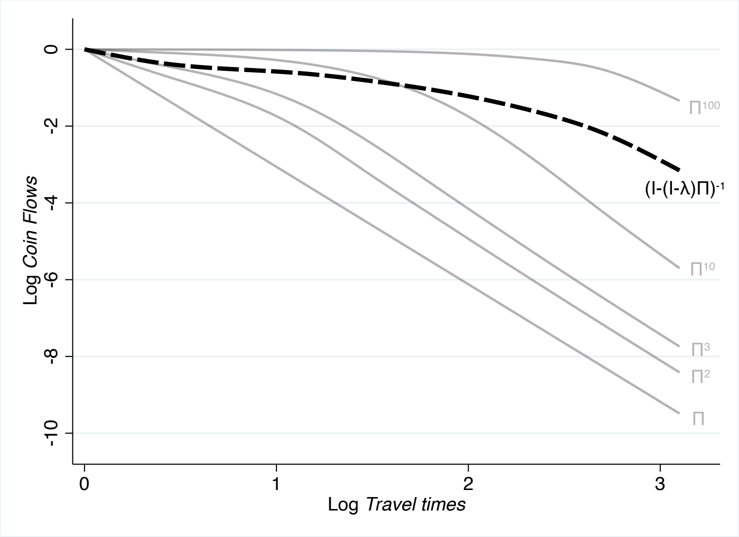
Coins vs trade (numerical example) ▸ Data



- Flow of coins: age 1, age 2, age 3, age 10, age 100

▸ back

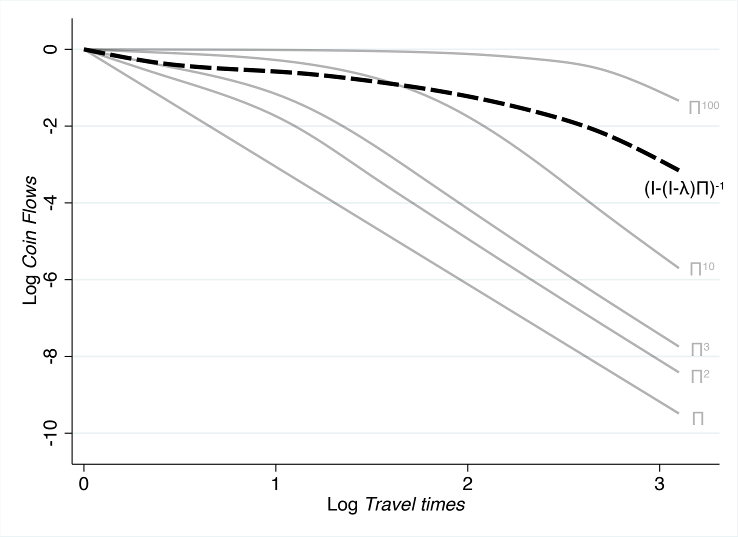
Coins vs trade (numerical example) ▸ Data



- Flow of coins: age 1, age 2, age 3, age 10, age 100, all ages

▸ back

Coins vs trade (numerical example) ▸ Data



- Flow of coins: age 1, age 2, age 3, age 10, age 100, all ages, as in Chaney (2018) ▸ back

From anonymous coins to specific coins (with saving)

- Stock $S_i[\tau]$ composed of different coin types, $S_i[\tau] = \sum_{m=1}^{N} \sum_{t \leq \tau} S_{mi}[t,\tau]$
- Coins start their 'coin life' when they are minted, $S_{mm}[t,t] = M_m[t]$
- Then stocks evolves recursively,

$$S_{mi}[t,\tau] = (1-\lambda)\left(s_i[\tau-1]\, S_{mi}[t,\tau-1] + \sum_{n=1}^{N} \pi_{ni}[\tau]\,(1-s_n[\tau-1])\, S_{mn}[t,\tau-1]\right)$$

- Recursive solution in matrix form,

$$\mathbf{S}[t,T] = (1-\lambda)^{T-t}\, \boldsymbol{M}[t] \times \tilde{\mathbf{\Pi}}[t+1]\, \tilde{\mathbf{\Pi}}[t+2] \cdots \tilde{\mathbf{\Pi}}[T]$$

- Dynamics driven by the coin flow matrix ($\tilde{\mathbf{\Pi}}$), not the trade matrix ($\mathbf{\Pi}$),

$$\tilde{\mathbf{\Pi}}[\tau] = (\boldsymbol{I} - \boldsymbol{s}[\tau])\, \mathbf{\Pi}[\tau] + \boldsymbol{s}[\tau]$$

- Saving magnifies the home bias.

Assumptions: (1) monetized economy + (2) coins are fungible

back

From anonymous coins to specific coins (with saving)

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$$S_{mi}[t, \tau] = (1 - \lambda) \left(s_i[\tau - 1] \, S_{mi}[t, \tau - 1] + \sum_{n=1}^{N} \pi_{ni}[\tau] \, (1 - s_n[\tau - 1]) \, S_{mn}[t, \tau - 1] \right)$$

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Assumptions: (1) monetized economy + (2) coins are fungible

back

Without saving, same old gravity up to a multiplicative constant

	Eaton and Kortum (2002) trade flows	Saving-'augmented' coin flows, $s_n = 0$
Trade/coin shares	$\pi_{ni} = \alpha_n \beta_i \delta_{ni}$	$\tilde{\pi}_{ni} = \tilde{\alpha}_n \tilde{\beta}_i \tilde{\delta}_{ni}$
Seller FE	$\beta_i = T_i (w_i)^{-\theta}$	$\tilde{\beta}_i = T_i (w_i)^{-\theta}$
Buyer FE	$\alpha_n = \frac{1}{\sum_k \beta_k \delta_{nk}}$	$\tilde{\alpha}_n = \frac{1}{\sum_k \tilde{\beta}_k \tilde{\delta}_{nk}}$
Bilateral (internal)	$\delta_{nn} = 1$	$\tilde{\delta}_{nn} = 1$
Bilateral (external)	$\delta_{ni} = \frac{(d_{ni})^{-\theta}}{(d_{nn})^{-\theta}}$	$\tilde{\delta}_{ni} = \frac{(d_{ni})^{-\theta}}{(d_{nn})^{-\theta}}$

- $\frac{\tilde{\delta}_{nj}}{\tilde{\delta}_{ni}} = \frac{(d_{nj})^{-\theta}}{(d_{ni})^{-\theta}}, \forall s_n \geq 0$: external flows depend on trade costs
- $\frac{\tilde{\delta}_{nn}}{\tilde{\delta}_{ni}} > \frac{(d_{nn})^{-\theta}}{(d_{ni})^{-\theta}}, \forall s_n > 0$: net savings mimics home bias!

Assumptions: (1) monetized economy + (2) coins are fungible + (3) EK trade

back

Without saving, same old gravity up to a multiplicative constant

	Eaton and Kortum (2002) trade flows	Saving-'augmented' coin flows, $s_n \geq 0$
Trade/coin shares	$\pi_{ni} = \alpha_n \beta_i \delta_{ni}$	$\tilde{\pi}_{ni} = \tilde{\alpha}_n \tilde{\beta}_i \tilde{\delta}_{ni}$
Seller FE	$\beta_i = T_i (w_i)^{-\theta}$	$\tilde{\beta}_i = T_i (w_i)^{-\theta}$
Buyer FE	$\alpha_n = \frac{1}{\sum_k \beta_k \delta_{nk}}$	$\tilde{\alpha}_n = \frac{1}{\sum_k \tilde{\beta}_k \tilde{\delta}_{nk}}$
Bilateral (internal)	$\delta_{nn} = 1$	$\tilde{\delta}_{nn} = 1$
Bilateral (external)	$\delta_{ni} = \frac{(d_{ni})^{-\theta}}{(d_{nn})^{-\theta}}$	$\tilde{\delta}_{ni} = \frac{(d_{ni})^{-\theta}}{(d_{nn})^{-\theta}} \left(1 - \frac{s_n[t]}{\tilde{\pi}_{nn}[t]}\right)$

- $\frac{\tilde{\delta}_{nj}}{\tilde{\delta}_{ni}} = \frac{(d_{nj})^{-\theta}}{(d_{ni})^{-\theta}}, \forall s_n \geq 0$: external flows depend on trade costs
- $\frac{\tilde{\delta}_{nn}}{\tilde{\delta}_{ni}} > \frac{(d_{nn})^{-\theta}}{(d_{ni})^{-\theta}}, \forall s_n > 0$: net savings mimics home bias!

Assumptions: (1) monetized economy + (2) coins are fungible + (3) EK trade

back

Regions

Back

Exponential decay of coin ages (tpq minus mint date)

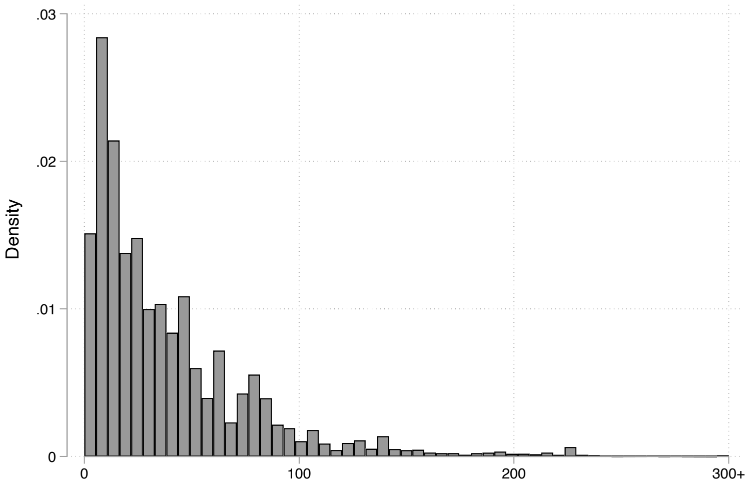

Figure: Coin age at time of deposit (*tpq*), in years

Back

Trade cost function

- *Assumption* (4) Parameterize trade cost (theory): $d_{nn}[t] = 1, \forall n, t$ and,

$$\ln\left((d_{ni}[t])^{-\theta}\right) = \gamma_0 + \zeta \ln\left(TravelTime_{ni}\right) + \kappa_1 PoliticalBorder_{ni}[t] + \kappa_2 ReligiousBorder_{ni}[t]$$

- $\Rightarrow$ bilateral determinants of coin flows (data): $\tilde{\delta}_{nn}[t] = 1, \forall n, t$ and,

$$\ln\left(\tilde{\delta}_{ni}[t]\right) = \tilde{\gamma}_0 + \zeta \ln\left(TravelTime_{ni}\right) + \kappa_1 PoliticalBorder_{ni}[t] + \kappa_2 ReligiousBorder_{ni}[t]$$

- From coin flows to trade flows (ancient saving rate into coins, 1.5% Scheidel, 2020),

$$\gamma_0 \ s.t. \ \left(1 - e^{\tilde{\gamma}_0 - \gamma_0}\right) \mathbb{E}_{n,t}\left[\tilde{\pi}_{nn}[t]\right] = \mathbb{E}_{n,t}\left[s_n[t]\right] = 0.015.$$

back

Trade cost function

- *Assumption* (4) Parameterize trade cost (theory): $d_{nn}[t] = 1, \forall n, t$ and,

$$\ln\left((d_{ni}[t])^{-\theta}\right) = \gamma_0 + \zeta \ln(TravelTime_{ni}) + \kappa_1 PoliticalBorder_{ni}[t] + \kappa_2 ReligiousBorder_{ni}[t]$$

- $\Rightarrow$ bilateral determinants of coin flows (data): $\tilde{\delta}_{nn}[t] = 1, \forall n, t$ and,

$$\ln\left(\tilde{\delta}_{ni}[t]\right) = \tilde{\gamma}_0 + \zeta \ln(TravelTime_{ni}) + \kappa_1 PoliticalBorder_{ni}[t] + \kappa_2 ReligiousBorder_{ni}[t]$$

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back

Trade cost function

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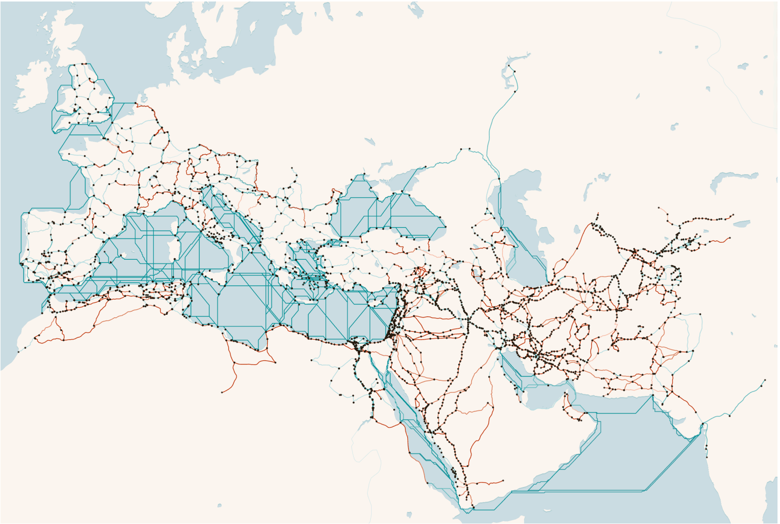
back

Optimal travel times with ancient technology

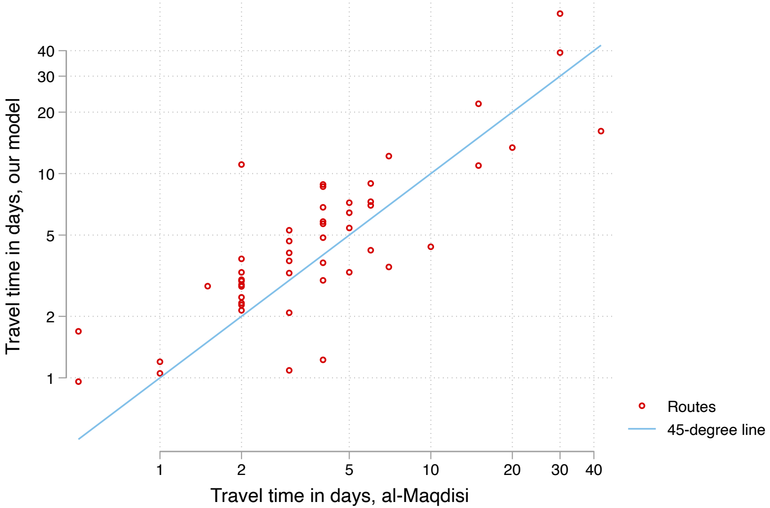


Note: Combined geospatial models from Orbis (Scheidel, 2015) and al-Ṯurayyā. ▸ al-Maqdisi

▸ back

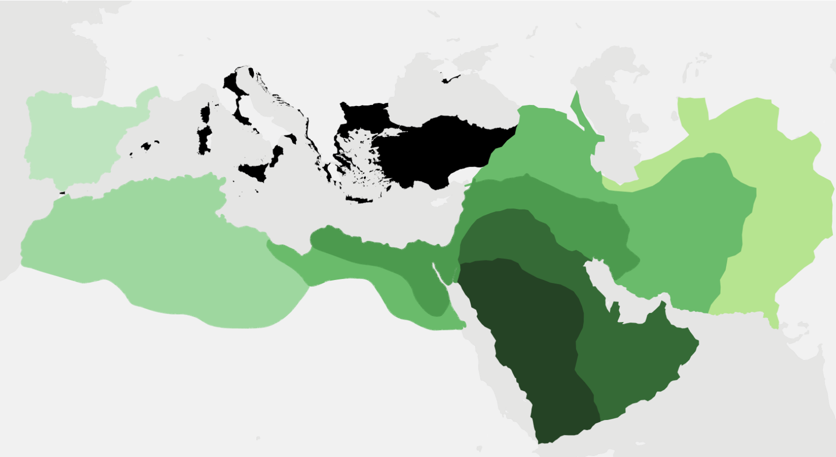
Validating Travel Times

Al-Maqdisi (c. 945–991): *The Best Divisions for Knowledge of the Regions*



Back

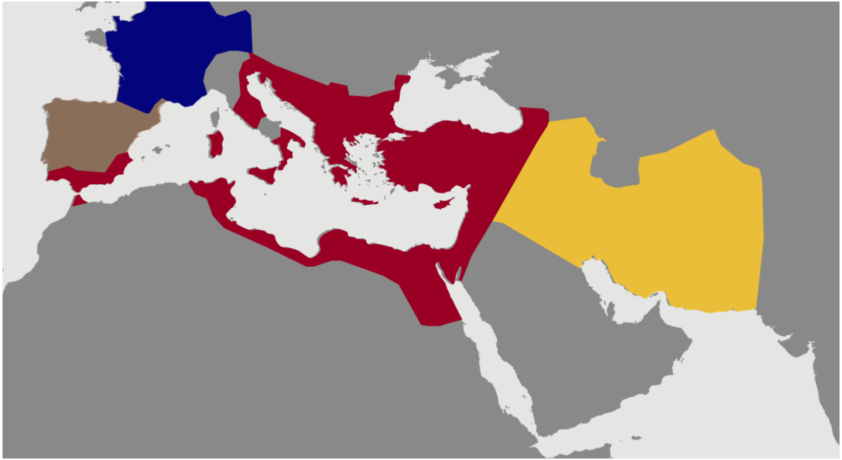
The Arab Conquests, AD 632-750



Byzantine Empire, 700	634	661	661-750
632, under Mohammed	644	661-700	700-750

▸ pre-Islam ▸ back

Pre-islamic ancient Western world, 600 AD



Visigoths Franks Byzantine Empire Sasanian Empire

Back

Technical steps to recover all equilibrium variables from parameters

1. Parameters: seller terms $T_n w_n^{-\theta}[t]$, trade costs $d_{ni}^{-\theta}[t]$, minting $M_n[t]$, coin exit rate λ
2. Trade shares:

$$\pi_{ni}[t] = \left(T_i w_i^{-\theta}[t]\right) d_{ni}^{-\theta}[t] / \sum_k \left(T_k w_k^{-\theta}[t]\right) d_{nk}^{-\theta}[t], \forall i, n, t$$

3. Nominal incomes (from market clearing):
   - 3a. initial period t_0, one single linear system of equations

$$w_i L_i[t_0] = \sum_n \pi_{ni}[t_0] \Big((1-\lambda) \, w_n L_n[t_0] + M_n[t_0] \Big), \forall i$$

   - 3b. subsequent periods $t+1 > t_0$

$$w_i L_i[t+1] = \sum_n \pi_{ni}[t+1] \Big((1-\lambda) \, w_n L_n[t] + M_n[t+1] \Big), \forall i, t \geq t_0$$

4. Tech-labor combo (assume trade elasticity $\theta = 4$):

$$L_i T_i^{1/\theta}[t] = (w_i L_i[t]) \left(T_i w_i^{-\theta}[t] \right)^{1/\theta}, \forall i, t$$

5. Technology (assume "Malthusian force" $L_n = T_n$):

$$T_i^{1/\theta}[t] = \left(L_i T_i^{1/\theta}[t] \right)^{\frac{1}{1+\theta}} \text{ and } L_i[t] = \left(L_i T_i^{1/\theta}[t] \right)^{\frac{\theta}{1+\theta}}, \forall i, t$$

back

Technical steps to compute counterfactual equilibrium (steady state)

1. Choose any counterfactual parameters
   (technology T_n, labor L_n, trade costs d_{ni}, minting M_n)
2. Solve for counterfactual wages $\boldsymbol{w_n}$ as fixed point:
   2a. Plug $(\mathtt{n})^{th}$ guess for wages $w_n^{(\mathtt{n})}$ to compute trade shares $\boldsymbol{\pi_{ni}^{(\mathtt{n})}}$

$$\boldsymbol{\pi_{ni}^{(\mathtt{n})}} = \frac{\left(T_i \left(w_i^{(\mathtt{n})}\right)^{-\theta}\right) d_{ni}^{-\theta}}{\sum_k \left(T_k \left(w_k^{(\mathtt{n})}\right)^{-\theta}\right) d_{nk}^{-\theta}}, \forall n, i$$

   2b. Plug trade shares $\pi_{ni}^{(\mathtt{n})}$ to update $(\mathtt{n+1})^{th}$ guess for wages $\boldsymbol{w_n^{(\mathtt{n+1})}}$ (linear system of equations)

$$\boldsymbol{w_i^{(\mathtt{n+1})}} L_i = \sum_n \pi_{ni}^{(\mathtt{n})} \left((1-\lambda)\, \boldsymbol{w_n^{(\mathtt{n+1})}} L_n + M_n\right), \forall i$$

   2c. Iterate until convergence (Alvarez and Lucas, 2007)

back

Realized vs counterfactual changes in real consumption

Realized changes, from AD 460-620 to AD 700-900

	Real consumption $\Delta \log\left(\frac{X_n/p_n}{L_n}\right)$		Openness $\Delta \log\left(\pi_{nn}^{-1/\theta}\right)$		Technology $\Delta \log\left(T_n^{1/\theta}\right)$		Trade Deficit $\Delta \log\left(1+\frac{M_n-\lambda w_n L_n}{w_n L_n}\right)$	
Francia and Germania	**1.96**	(0.24)	**-0.05**	(0.01)	**1.80**	(0.26)	**0.20**	(0.04)
Byzantine Heartlands	**-1.56**	(0.33)	**-0.23**	(0.14)	**-0.44**	(0.41)	**-0.89**	(0.54)
Arabian Peninsula	**1.16**	(0.34)	**-0.01**	(0.04)	**0.66**	(0.45)	**0.51**	(0.26)

Counterfactual changes relative to to AD 700-900

	Realized $\Delta \log\left(\frac{X_n/p_n}{L_n}\right)$		Counterfactual $\Delta \log\left(\frac{X_n/p_n}{L_n}\right)$ if:					
	All parameters AD 700-900		Religious border AD 700-900		Technology AD 700-900		Minting AD 700-900	
Francia and Germania	**1.96**	(0.24)	**-0.07**	(0.02)	**1.68**	(0.11)	**6.17**	(0.47)
Byzantine Heartlands	**-1.56**	(0.33)	**-0.69**	(0.08)	**-0.57**	(0.13)	**-1.41**	(0.19)
Arabian Peninsula	**1.16**	(0.34)	**0.26**	(0.09)	**0.66**	(0.40)	**2.71**	(0.84)

Normalizations: realized $\Delta \log\left(\mathbb{E}\left[\frac{X_n/p_n}{L_n}\right]\right) \equiv 0$. Bootstrapped s.e.'s in parentheses (100 bootstraps).

back

Realized vs counterfactual changes in real consumption

Realized changes, from AD 460-620 to AD 700-900

	Real consumption $\Delta \log \left(\frac{X_n/p_n}{L_n}\right)$		Openness $\Delta \log \left(\pi_{nn}^{-1/\theta}\right)$		Technology $\Delta \log \left(T_n^{1/\theta}\right)$		Trade Deficit $\Delta \log \left(1 + \frac{M_n - \lambda w_n L_n}{w_n L_n}\right)$	
al-Andalus (Spain)	0.62	(0.25)	-0.06	(0.04)	0.77	(0.32)	-0.09	(0.18)
Aquitaine (South France)	1.28	(0.23)	-0.05	(0.01)	1.22	(0.23)	0.11	(0.06)
Francia and Germania	**1.96**	**(0.24)**	**-0.05**	**(0.01)**	**1.80**	**(0.26)**	**0.20**	**(0.04)**
Northern Italy	-0.31	(0.24)	-0.08	(0.03)	-0.10	(0.26)	-0.13	(0.10)
Southern Italy	-0.20	(0.34)	0.19	(0.18)	-0.94	(0.37)	0.55	(0.40)
Byzantine Heartlands	**-1.56**	**(0.33)**	**-0.23**	**(0.14)**	**-0.44**	**(0.41)**	**-0.89**	**(0.54)**
al-Sham (Greater Syria)	-0.32	(0.27)	-0.04	(0.02)	-0.11	(0.29)	-0.17	(0.11)
Northern Syria, Caucasus	0.22	(0.30)	-0.01	(0.03)	0.15	(0.37)	0.08	(0.12)
Iraq, Iran	0.06	(0.27)	-0.00	(0.01)	0.06	(0.29)	-0.00	(0.04)
Eastern Caliphate	0.37	(0.33)	-0.00	(0.00)	0.39	(0.34)	-0.02	(0.04)
Arabian Peninsula	**1.16**	**(0.34)**	**-0.01**	**(0.04)**	**0.66**	**(0.45)**	**0.51**	**(0.26)**
Misr (Egypt)	-0.36	(0.72)	0.09	(0.23)	-0.82	(0.50)	0.37	(0.90)
al-Maghrib	0.28	(0.33)	0.13	(0.07)	-0.49	(0.27)	0.65	(0.30)

Normalizations: $\Delta \log \left(\mathbb{E}\left[\frac{X_n/p_n}{L_n}\right]\right) \equiv 0$. Bootstrapped s.e.'s in parentheses (100 bootstraps).

back

Realized vs counterfactual changes in real consumption

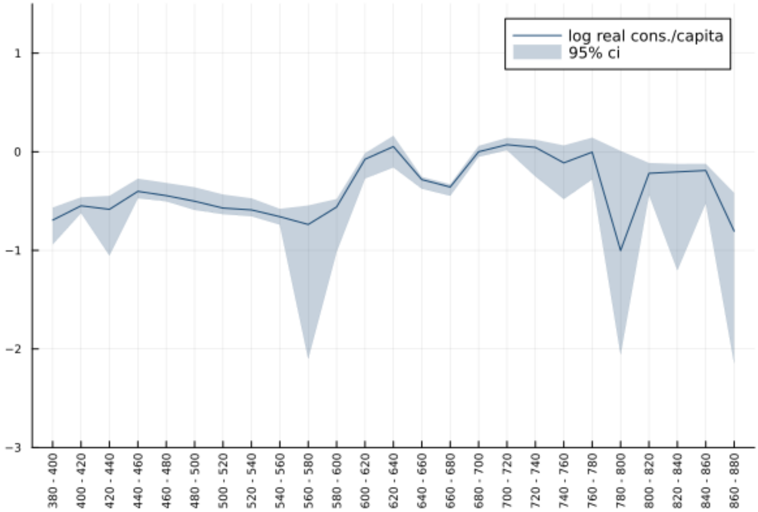
Counterfactual changes relative to to AD 700-900

	Initial $\log\left(\frac{X_n/p_n}{L_n}\right)$		Counterfactual $\Delta \log\left(\frac{X_n/p_n}{L_n}\right)$ if:					
	All parameters AD 460-620		Religious border AD 700-900		Technology AD 700-900		Minting AD 700-900	
al-Andalus (Spain)	-0.70	(0.10)	0.09	(0.02)	0.55	(0.10)	1.57	(0.31)
Aquitaine (South France)	-1.04	(0.08)	-0.15	(0.03)	0.99	(0.09)	3.93	(0.30)
Francia and Germania	**-1.55**	**(0.09)**	**-0.07**	**(0.02)**	**1.68**	**(0.11)**	**6.17**	**(0.47)**
Northern Italy	0.07	(0.04)	-0.24	(0.05)	-0.24	(0.08)	-0.21	(0.07)
Southern Italy	-0.25	(0.06)	-0.11	(0.02)	-0.60	(0.13)	-0.03	(0.02)
Byzantine Heartlands	**1.22**	**(0.11)**	**-0.69**	**(0.08)**	**-0.57**	**(0.13)**	**-1.41**	**(0.19)**
al-Sham (Greater Syria)	0.30	(0.04)	0.04	(0.01)	-0.18	(0.10)	-0.22	(0.08)
Northern Syria, Caucasus	-0.34	(0.11)	0.02	(0.02)	0.15	(0.22)	0.19	(0.19)
Iraq, Iran	0.28	(0.08)	0.01	(0.00)	0.03	(0.08)	0.03	(0.06)
Eastern Caliphate	-0.44	(0.08)	0.01	(0.00)	0.38	(0.16)	0.34	(0.26)
Arabian Peninsula	**-1.80**	**(0.18)**	**0.26**	**(0.09)**	**0.66**	**(0.40)**	**2.71**	**(0.84)**
Misr (Egypt)	0.32	(0.07)	0.02	(0.00)	-0.71	(0.24)	-0.09	(0.02)
al-Maghrib	0.12	(0.06)	0.01	(0.00)	-0.46	(0.17)	-0.05	(0.06)

Normalizations: initial $\log\left(\mathbb{E}\left[\frac{X_n/p_n}{L_n}\right]\right) \equiv 0$. Bootstrapped s.e.'s in parentheses (100 bootstraps).

back

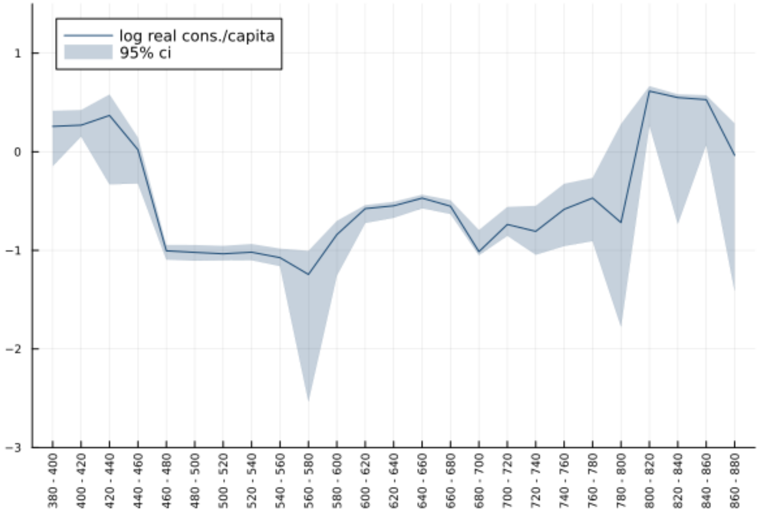
Real consumption per capita (380-880): al-Andalus (Spain)



Bootstrapped 95% confidence intervals. *Normalizations:* $\ln\left(\mathbb{E}_t\left[\frac{X_n[t]/p_n[t]}{L_n[t]}\right]\right) \equiv 0, \forall t.$

back

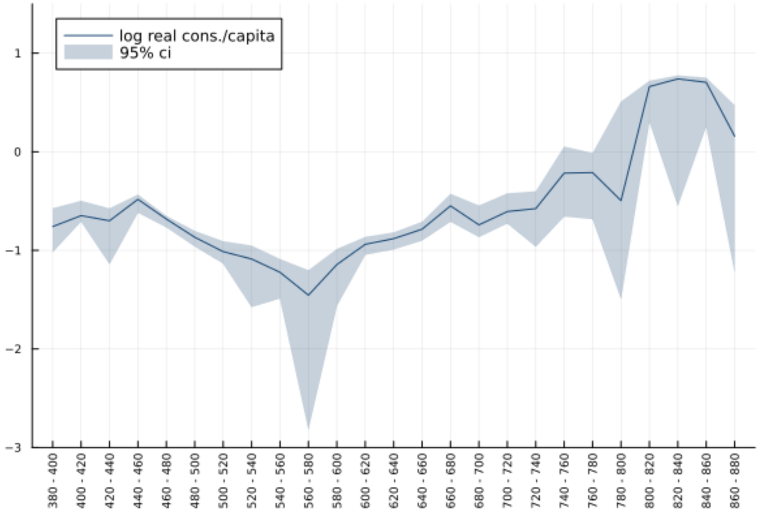
Real consumption per capita (380-880): Aquitaine (South France)



Bootstrapped 95% confidence intervals. *Normalizations:* $\ln\left(\mathbb{E}_t\left[\frac{X_n[t]/p_n[t]}{L_n[t]}\right]\right) \equiv 0, \forall t.$

back

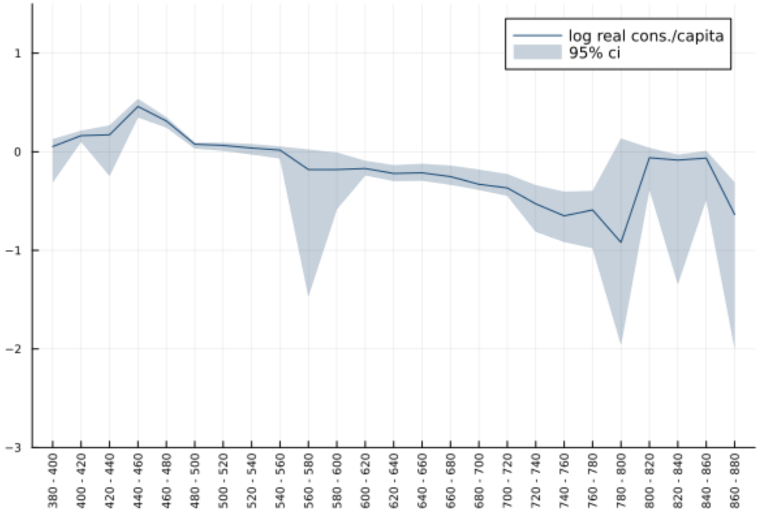
Real consumption per capita (380-880): Francia and Germania



Bootstrapped 95% confidence intervals. *Normalizations:* $\ln\left(\mathbb{E}_t\left[\frac{X_n[t]/p_n[t]}{L_n[t]}\right]\right) \equiv 0, \forall t.$

back

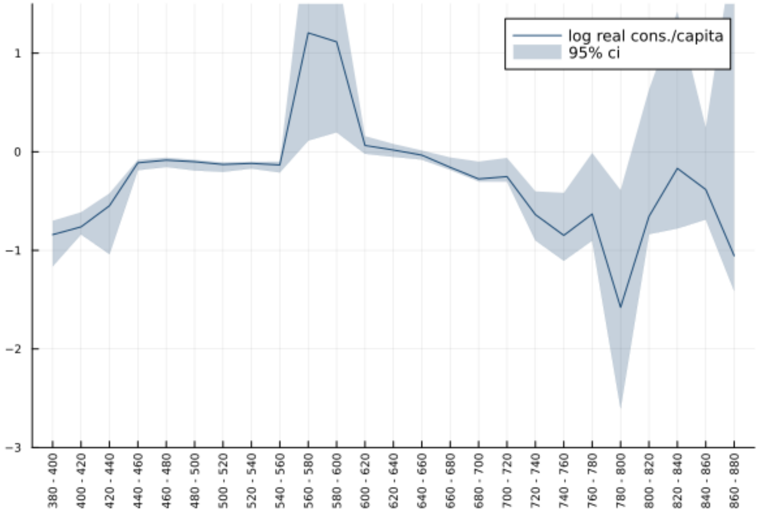
Real consumption per capita (380-880): Northern Italy



Bootstrapped 95% confidence intervals. *Normalizations:* $\ln\left(\mathbb{E}_t\left[\frac{X_n[t]/p_n[t]}{L_n[t]}\right]\right) \equiv 0, \forall t.$

back

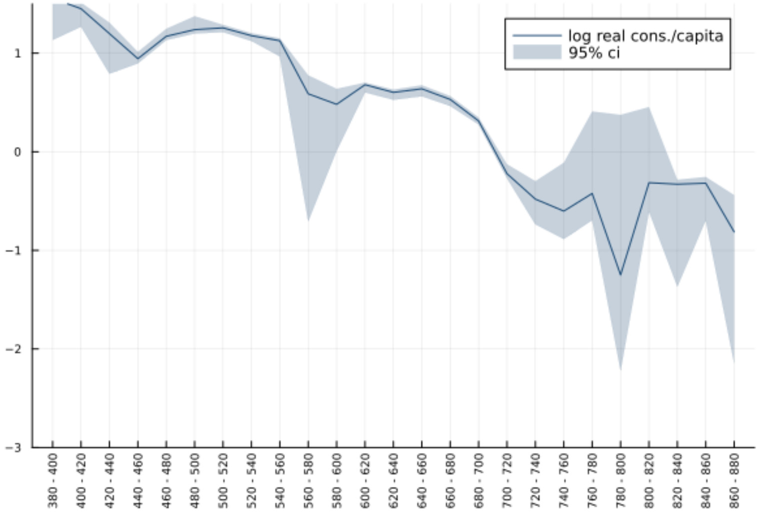
Real consumption per capita (380-880): Southern Italy



Bootstrapped 95% confidence intervals. *Normalizations:* $\ln\left(\mathbb{E}_t\left[\frac{X_n[t]/p_n[t]}{L_n[t]}\right]\right) \equiv 0, \forall t.$

▸ back

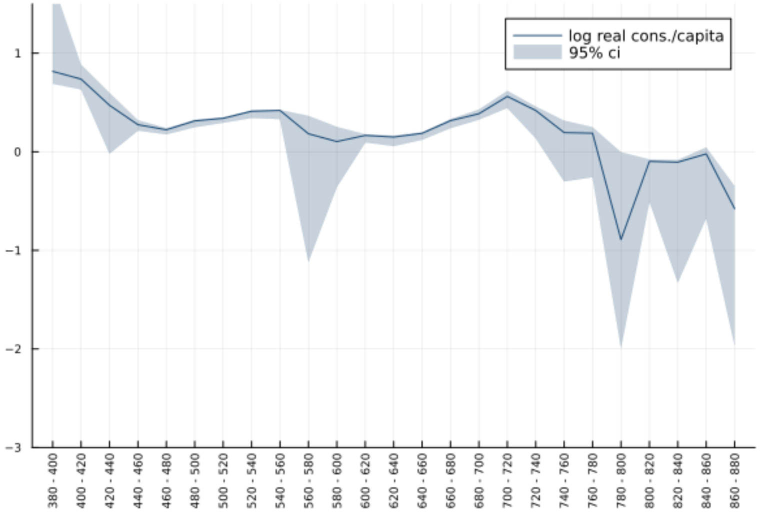
Real consumption per capita (380-880): Byzantine Heartlands



Bootstrapped 95% confidence intervals. *Normalizations:* $\ln\left(\mathbb{E}_t\left[\frac{X_n[t]/p_n[t]}{L_n[t]}\right]\right) \equiv 0, \forall t.$

back

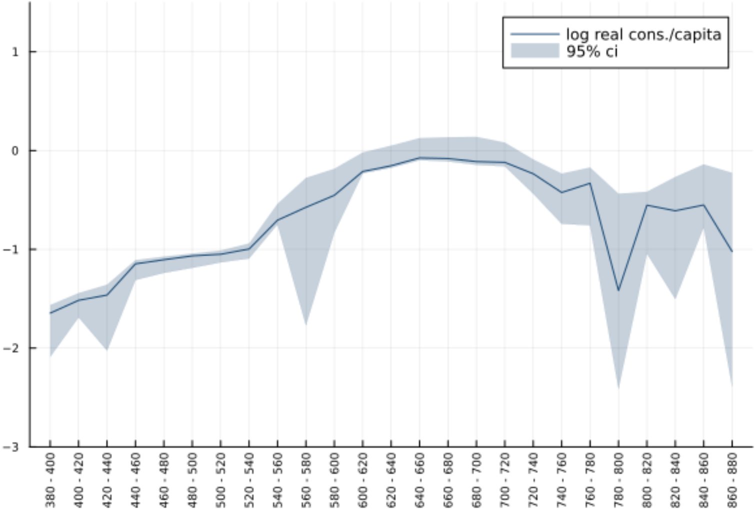
Real consumption per capita (380-880): al-Sham (Greater Syria)



Bootstrapped 95% confidence intervals. *Normalizations:* $\ln\left(\mathbb{E}_t\left[\frac{X_n[t]/p_n[t]}{L_n[t]}\right]\right) \equiv 0, \forall t.$

back

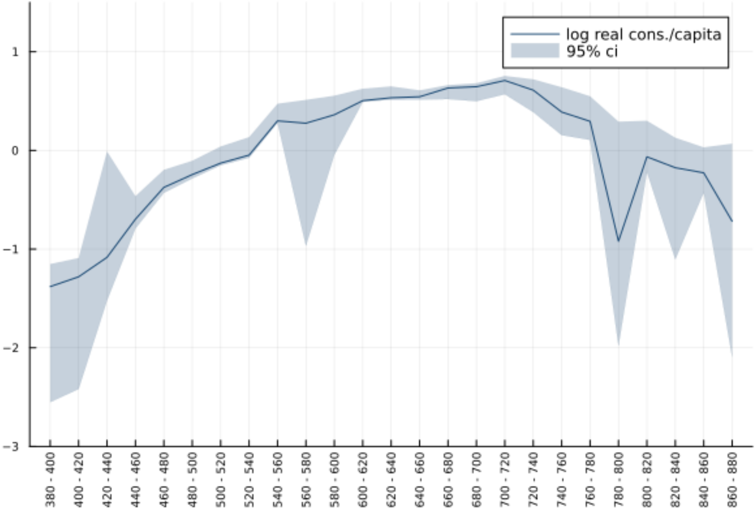
Real consumption per capita (380-880): Northern Syria, Caucasus



Bootstrapped 95% confidence intervals. *Normalizations:* $\ln\left(\mathbb{E}_t\left[\frac{X_n[t]/p_n[t]}{L_n[t]}\right]\right) \equiv 0, \forall t.$

back

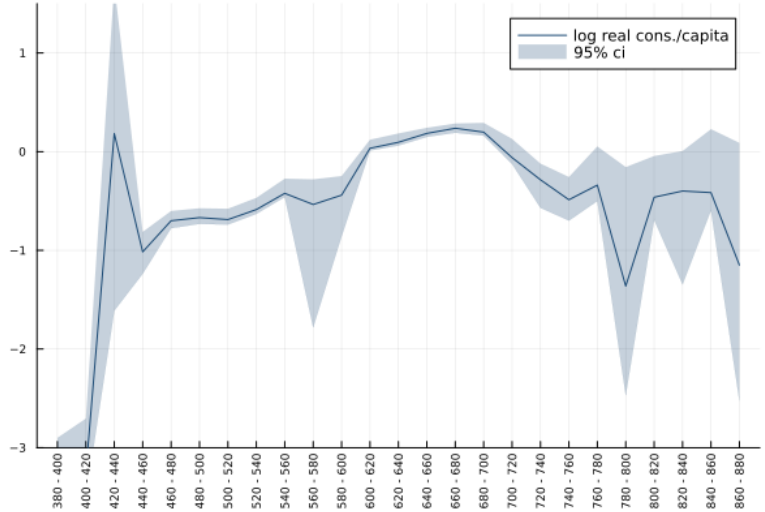
Real consumption per capita (380-880): Iraq, Iran



Bootstrapped 95% confidence intervals. *Normalizations:* $\ln\left(\mathbb{E}_t\left[\frac{X_n[t]/p_n[t]}{L_n[t]}\right]\right) \equiv 0, \forall t.$

back

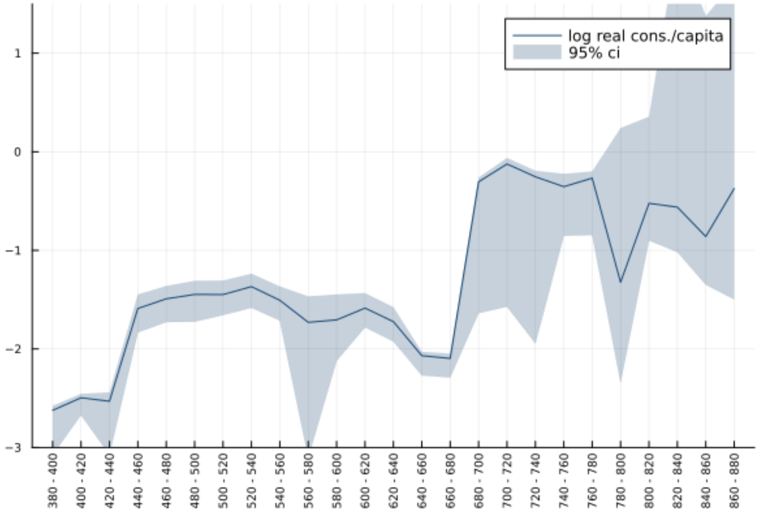
Real consumption per capita (380-880): Eastern Caliphate



Bootstrapped 95% confidence intervals. *Normalizations:* $\ln\left(\mathbb{E}_t\left[\frac{X_n[t]/p_n[t]}{L_n[t]}\right]\right) \equiv 0, \forall t.$

back

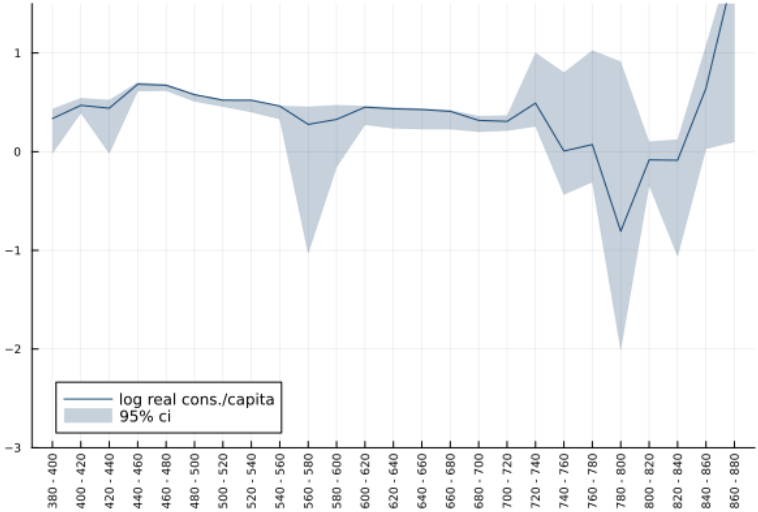
Real consumption per capita (380-880): Arabian Peninsula



Bootstrapped 95% confidence intervals. *Normalizations:* $\ln\left(\mathbb{E}_t\left[\frac{X_n[t]/p_n[t]}{L_n[t]}\right]\right) \equiv 0, \forall t.$

back

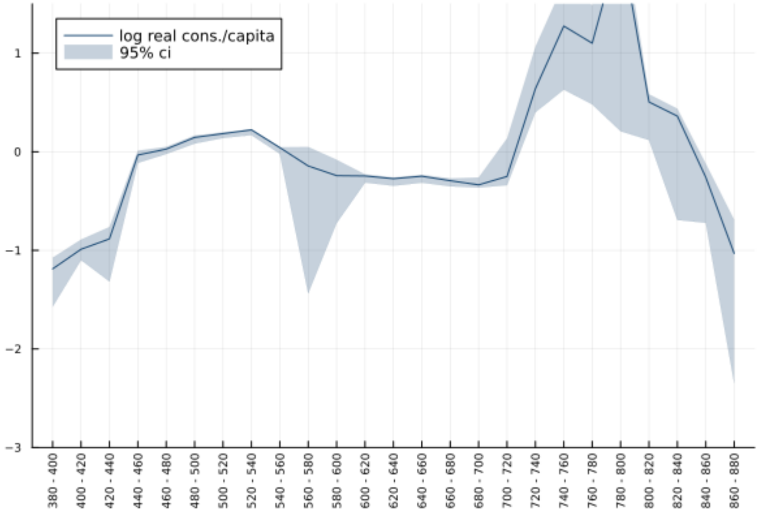
Real consumption per capita (380-880): Misr (Egypt)



Bootstrapped 95% confidence intervals. *Normalizations:* $\ln\left(\mathbb{E}_t\left[\frac{X_n[t]/p_n[t]}{L_n[t]}\right]\right) \equiv 0, \forall t.$

back

Real consumption per capita (380-880): al-Maghrib



Bootstrapped 95% confidence intervals. *Normalizations:* $\ln\left(\mathbb{E}_t\left[\frac{X_n[t]/p_n[t]}{L_n[t]}\right]\right) \equiv 0, \forall t.$

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