

Technical Appendix for:

THE GRAVITY EQUATION IN INTERNATIONAL TRADE: AN EXPLANATION  
(not for publication)

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This technical appendix is composed of four parts. In parts **A** and **B**, I build an explicit micro-founded economic model that generates endogenously the conditions assumed for the model of firm to firm trade in section **3**. Formally, firms' sizes, the size of firms' shipments, and the parameters  $\gamma$  and  $\beta$ , are derived endogenously ( $\delta$  and  $K_0$  remain exogenous technological parameters). I also analyze the welfare consequences of a trade disruption in this model, both in the short run, and in the medium and long run. In part **C**, I present robustness checks and additional empirical evidence in support of proposition **1**. In part **D**, I present additional mathematical proofs.

## **A Modeling input-output linkages between plants**

I present an explicit economic model where the assumptions of the model in section **3** emerge endogenously. This section develops a simple dynamic model of input-output linkages between firms. Firms produce differentiated goods combining equipped labor and differentiated inputs produced by other firms. Over time, as a firm acquires new upstream suppliers and new downstream customers, it becomes more productive, and its size grows. I characterize a balanced growth path along which existing firms grow, new firms enter continuously, and the distribution of firm sizes is stationary and Pareto. But I abstract mostly from geography. Section **3** in the main paper embeds this model explicitly into geographic space.

### **A.1 Set-up**

There are two types of goods: final goods and intermediate inputs. Final goods are produced by combining differentiated intermediate inputs available locally, and are sold locally to consumers on a perfectly competitive market, which can be interpreted as a local retail sector. Intermediate inputs are themselves produced by combining differentiated inputs and equipped labor, so that the economy features roundabout production. They are produced and distributed worldwide by monopolistically competitive firms. Since only intermediate inputs are traded, I focus most of my

attention on the production and trade of intermediates. Due to matching frictions, intermediate input firms source their inputs from, and sell their output to a subset of producers only.

*Final goods.*— Final goods are sold on a competitive market by atomistic firms that share the same constant returns production function,

$$Q_{\text{final}} = \left( \int_{i \in \mathcal{S}} q_{i \rightarrow \text{final}}^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} \quad (15)$$

where  $\mathcal{S}$  is the set of local suppliers to the final assembler,  $q_{i \rightarrow \text{final}}$  is the quantity of inputs sold by firm  $i$  to the final assembler, and  $\sigma > 1$  is the elasticity of substitution between inputs. The final assembly sector can be thought of as a local distribution sector that offers goods locally to final consumers, or simply as the preference of local consumers.

*Intermediate inputs.*— Firm  $i$  buys intermediate inputs from a continuum of suppliers  $k \in \mathcal{S}_i$  and sells its output to a continuum of customers  $j \in \mathcal{C}_i$  as well as to the final good producers on a monopolistically competitive market. The customers  $j \in \mathcal{C}_i$  are themselves other intermediate input producers. Both the set of suppliers  $\mathcal{S}_i$  and the set of customers  $\mathcal{C}_i$  will be endogenously determined dynamically below. Firm  $i$  combines  $q_{k \rightarrow i}$  units of differentiated intermediate inputs from each supplier  $k \in \mathcal{S}_i$  ( $q_{k \rightarrow i}$  stands for the sales from  $k$  to  $i$ ) with  $L_i$  units of equipped labor to produce  $Q_i$  units of output,

$$Q_i = \left( \int_{k \in \mathcal{S}_i} q_{k \rightarrow i}^{\frac{\sigma-1}{\sigma}} dk \right)^{\alpha \frac{\sigma}{\sigma-1}} L_i^{1-\alpha} \quad (16)$$

with  $0 < \alpha < 1$  the share of intermediate inputs in production and  $\sigma > 1$  the elasticity of substitution between any two intermediate inputs.

*Free entry.*— There is a perfectly elastic fringe of potential entrepreneurs ready to start an intermediate input producing firms. Starting up a business entails a constant fixed entry cost of  $f_E$  units of labor. This free entry of new firms ensures that in equilibrium, all profits are dissipated.

*Workers and consumers.*— In every location, there is a competitive labor market with an equilibrium wage  $w$ , and a measure  $L_t$  of consumers/workers at time  $t$ . Population grows at a constant rate  $\gamma_L$ . I will consider only symmetric equilibria where wages and population are the same across locations.

## A.2 The static problem of the firm

Consider what happens within period  $t$ . For the moment, I drop the time  $t$  index.

*Firm demand.*— Each period, firm  $i$  faces the same iso-elastic demand from any customer, whether they are another intermediate good producer or the final good producer,  $j \in \{\mathcal{C}_i \cup \text{final}\}$ ,

$$p_{i \rightarrow j} q_{i \rightarrow j} = \frac{p_{i \rightarrow j}^{1-\sigma}}{\int_{k \in \mathcal{S}_j} p_{k \rightarrow j}^{1-\sigma} dk} X_j \quad (17)$$

with  $p_{i \rightarrow j}$  the price charged by  $i$  to customer  $j$ ,  $q_{i \rightarrow j}$  the units sold by  $i$  to  $j$ ,  $\mathcal{S}_j$  the set of suppliers of  $j$ , and  $X_j$  the total spending on intermediate inputs by  $j$ .

*Firm pricing.*— Given these iso-elastic demands, firm  $i$  charges all its customers the same constant mark-up,  $\frac{\sigma}{\sigma-1}$ , over its marginal cost,

$$p_{i \rightarrow j} = p_i = \frac{\sigma}{\sigma-1} w^{1-\alpha} \left( \int_{k \in \mathcal{S}_i} p_k^{1-\sigma} dk \right)^{\frac{\alpha}{1-\sigma}} \quad (18)$$

with  $w$  the competitive wage rate. Equation (18) sheds light on the complex interactions between firms' in a network of input-output linkages. The price firm  $i$  charges to any of its clients ( $p_i$ ) depends on both the prices charged by its suppliers (the  $p_k$ 's) and the diversity of its suppliers (the set  $\mathcal{S}_i$ ). With iso-elastic demand, prices are simply proportional to a firm's productivity. So from Equation (18), a firm with either more efficient suppliers (lower  $p_k$ 's) or with a more diverse set of suppliers (a larger measure for  $\mathcal{S}_i$ ) will have a lower marginal cost of production. But the productivity of each of firm  $i$ 's suppliers in turn depends on the efficiency of their suppliers, so that the structure of the entire upstream production chain matters for firm  $i$ 's efficiency.

*Firm sales.*— A firm  $i$  sells its output to its customers ( $j \in \mathcal{C}_i$ ), and to the final good producer. When selling to customer  $j$ , firm  $i$  competes against the other suppliers of  $j$  ( $k \in \mathcal{S}_j$ ). When selling to the final good producer, firm  $i$  competes against all local intermediate input producers ( $n \in \mathcal{S}$ ). The total sales of firm  $i$  are,

$$p_i Q_i = p_i^{1-\sigma} \int_{\mathcal{C}_i} \frac{X_j}{\int_{\mathcal{S}_j} p_k^{1-\sigma} dk} dj + p_i^{1-\sigma} \frac{wL}{P^{1-\sigma}} \quad (19)$$

where  $X_j$  is the total spending on intermediate inputs by customer  $j$ ,  $P = \left( \int_{\mathcal{S}} p_n^{1-\sigma} dn \right)^{\frac{1}{1-\sigma}}$  is the ideal price index for all intermediate goods purchased by the local final good producer, and  $wL$  is the total spending on intermediate inputs by the final good producer.<sup>17</sup>

From Equation (19) firm  $i$ 's output, sales, and ultimately profits, which are a constant fraction  $1/\sigma$  of total sales, increase with the number of  $i$ 's customers (the measure of  $\mathcal{C}_i$ ). The measure of

<sup>17</sup>As the final goods market is competitive, the total spending on intermediate inputs by the final goods producer is equal to the income of the local consumers. Because of free entry, all profits are dissipated, and aggregate income equals  $wL$ . A priori, aggregate spending  $wL$  as well as the price index  $P$  may differ across locations, but in the symmetric equilibrium I consider, they will be equalized across locations.

customers plays a role equivalent to a demand shifter. From Equation (18), firm  $i$ 's output, sales and profits increase with the number of  $i$ 's suppliers (the measure of  $\mathcal{S}_i$ ). The measure of suppliers plays a role equivalent to capital, or to a productivity shifter. The notion that the diversity of intermediate inputs plays a role similar to productivity or capital has been explored since at least Romer (1990).<sup>18</sup> Because of these two forces, in the presence of informational frictions that prevent a firm from buying and selling to all other firms in the economy, firm  $i$  is willing to pay to acquire information about upstream suppliers and downstream customers. I introduce an explicit market for information in section A.4, so that the sets of customers ( $\mathcal{C}_i$ ) and suppliers ( $\mathcal{S}_i$ ) are determined endogenously over time. Before solving for this dynamic choice of the firm, I first need to characterize the general equilibrium each period, which I turn to now.

### A.3 The static general equilibrium

Having characterized the static behavior of firms, I solve for the general equilibrium each period.

*Symmetric equilibrium.*— For simplicity, I will consider a symmetric equilibrium where all firms within a connected network of suppliers and customers have the same measure of suppliers and of customers. Formally, firm  $i$  belongs to a production network  $\mathcal{G}_i$  defined as,

$$\mathcal{G}_i = \mathcal{C}_i \cup \mathcal{S}_i \cup (\cup_{j \in \mathcal{C}_i} \mathcal{C}_j \cup \mathcal{S}_j \cup (\dots)) \cup (\cup_{k \in \mathcal{S}_i} \mathcal{C}_k \cup \mathcal{S}_k \cup (\dots))$$

The production network  $i$  belongs to,  $\mathcal{G}_i$ , is a set which contains all  $i$ 's suppliers ( $\mathcal{S}_i$ ) and all of  $i$ 's customers ( $\mathcal{C}_i$ ); for any of  $i$ 's customer  $j \in \mathcal{C}_i$ , it also contains all of  $j$ 's suppliers ( $\mathcal{S}_j$ ) and all of  $j$ 's customers ( $\mathcal{C}_j$ ); for any of  $i$ 's supplier  $k \in \mathcal{S}_i$ , it also contains all of  $k$ 's suppliers ( $\mathcal{S}_k$ ) and all of  $k$ 's customers ( $\mathcal{C}_k$ ); ... etc. In simpler words, the set  $\mathcal{G}_i$  contains all the firms that are connected to  $i$  via supplier or customer links, directly or indirectly.

I consider a symmetric equilibrium such that all the firms in  $i$ 's production network have the same measure of suppliers and of customers as  $i$ , which I denote by  $K_i$ . It stand for “contacts” and plays a role analogous to “capital.” Formally I conjecture that in equilibrium,

$$||\mathcal{S}_j|| = ||\mathcal{C}_j|| = K_i, \forall j \in \mathcal{G}_i \tag{20}$$

Under this symmetry condition,<sup>19</sup> the pricing and the market shares of firms are easy to solve for.

From the pricing Equation (18), the price of firm  $i$  depends on the price of all of its suppliers, which

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<sup>18</sup>See among many examples the theoretical model of di Giovanni and Levchenko (2010) or the empirical evidence of Halpern, Koren and Szeidl (2011) for two recent applications of this notion in trade, or Jones (2011) for a recent application to growth and development.

<sup>19</sup>I show in the next section this symmetry condition holds as long as newborn firms form upstream and downstream links only with other similar newborn firms.

in turn depends on the prices of their suppliers,... etc. Prices are the solution to a potentially complex fixed point problem. Under symmetry, the distribution of prices of  $i$ 's suppliers no longer matters for  $i$ 's price, only the measure  $K_i$  does. Similarly, from the demand Equation (17), the market share that  $i$  commands to any of its customers depends on  $i$ 's own price, but also on the price of its competitors, which themselves depend on the prices of their suppliers,... etc. Under symmetry, all those prices are the same, and  $i$ 's market share is simply  $1/K_i$ .

The symmetry condition (20) is strong. It simplifies greatly the analysis of the optimal strategies of firms, of the equilibrium, and of the patterns of firm level and aggregate trade flows. Relaxing this condition forces me to keep track of the joint distribution of suppliers and customers of all firms in every location, and renders the analysis complex. I show in this appendix how to relax this condition. The most important predictions of this model (equations (11) and (12) in proposition 3) are preserved, but at a great cost in terms of analytical complexity. While the simplicity of the analysis it affords is the primary justification for this symmetry condition, it is also a somewhat appropriate approximation of reality: Atalay, Hortacsu, Roberts and Syverson (2011) show for the US a tendency towards positive assortative matching, whereby firms with many suppliers tend to be connected to suppliers with many suppliers themselves; Acemoglu, Carvalho, Ozdaglar and Tahbaz-Salehi (2012) show a similar property at the level of industrial sectors. Note also that while there is no heterogeneity between firms within a production chain, there is heterogeneity between firms in different production chains. Moreover, the network of input-output linkages within a production network is complex and will evolve in a complex fashion over time.

The symmetry condition (20) seems a good compromise between tractability and keeping rich predictions for firm heterogeneity.

*Equilibrium firm price.*— Each firm charges a price to any of its consumer that is a constant mark-up  $\frac{\sigma}{\sigma-1}$  over its marginal cost of production, which depends on the price charged by the firm's suppliers,... etc. Under the symmetry condition (20), a firm with  $K$  suppliers buys from suppliers with  $K$  suppliers themselves,... etc, all of which charge the same price  $p_K$ . Using the pricing Equation (18), I solve for the unit price  $p_K$  of a firm with  $K$  suppliers,

$$p_K = \frac{\sigma}{\sigma-1} w^{1-\alpha} \left( \int_K p_K^{1-\sigma} dk \right)^{\frac{\alpha}{1-\sigma}} \Rightarrow p_K^{1-\sigma} = K^{\frac{\alpha}{1-\alpha}} \quad (21)$$

where I normalize the competitive wage  $w$  so that  $\frac{\sigma}{\sigma-1} w^{1-\alpha} = 1$ . A firm with more suppliers is more efficient and charges a lower price. The more the diversity of suppliers matters for efficiency (the lower  $\sigma$ ) or the larger the multiplier effect of intermediate inputs (the larger  $\alpha$ ), the larger

the efficiency advantage of having more suppliers.

*Equilibrium aggregate prices.*— Given individual prices in Equation (21), if there is a measure  $M$  of firms in each location and  $F$  is the cumulative distribution of firms with  $K$  suppliers, then the ideal price index is given by,

$$P = \left( \int_{K_0}^{\infty} p_K^{1-\sigma} M dF(K) \right)^{\frac{1}{1-\sigma}} = \left( M \mathbb{E}_F \left[ K^{\frac{\alpha}{1-\alpha}} \right] \right)^{\frac{1}{1-\sigma}} \quad (22)$$

In a symmetric equilibrium, this price index is the same across locations. I solve endogenously for the measure of firms  $M$  in Section A.6 where I characterize the optimal entry decision of firms. I derive an explicit solution for the distribution  $F$  and prove that it is time invariant in Section 3.

Several observations are in order. First, the aggregate price index decreases with the measure of firms, as in any model with a love for variety. Second, more efficient firms (lower price/higher  $K$  firms) contribute more to lowering the aggregate price index, as in any model with heterogeneous firms. The novel aspect of this model is firms with lower prices are typically firms with many suppliers, part of more complex production chains. More complex and longer production chains contribute to more efficient production, and a lower aggregate price index.

*Equilibrium sales and shipments.*— Firm  $i$  sells its output to the final assembler and to each customer  $j \in \mathcal{C}_i$ . Each customer  $j$  charges a constant mark-up  $\frac{\sigma}{\sigma-1}$  over marginal cost, spends a constant fraction  $\alpha$  on intermediates inputs, so its total spending on inputs is a constant fraction of its sales,  $X_j = \alpha \frac{\sigma-1}{\sigma} p_j Q_j$ . In a symmetric equilibrium, if firm  $i$  has  $K$  suppliers, then each of its customers also has  $K$  suppliers, and spends the same amount on intermediates as  $i$  does. This allows me to solve for the total sales of a firm with  $K$  suppliers,

$$\text{Sales}(K) = K^{\frac{\alpha}{1-\alpha}} \left( \frac{wL/P^{1-\sigma}}{1 - \alpha \frac{\sigma-1}{\sigma}} \right) \quad (23)$$

and for the value of any individual shipment of a firm with  $K$  suppliers to any of its customer,

$$\text{Shipment}(K) = K^{\frac{2\alpha-1}{1-\alpha}} \left( \frac{wL/P^{1-\sigma}}{\frac{\sigma}{\alpha(\sigma-1)} - 1} \right) \quad (24)$$

Firms with more suppliers are more efficient at producing, they sell to more other intermediate input producers, they charge a lower price and are able to capture a larger market share of the final demand, and ultimately have larger total sales. When the share of intermediates is larger than that of equipped labor ( $\alpha > \frac{1}{2}$ ), the multiplier effect of intermediates is large enough so that size increases more than proportionately with the number of suppliers  $K$ , and individual shipments

sizes increase with  $K$ . When  $\alpha < \frac{1}{2}$ , the opposite happens. In the special case where the share of intermediates equals the share of equipped labor ( $\alpha = \frac{1}{2}$ ) the total sales of a firm with  $K$  suppliers are exactly proportional to  $K$ , all firms send shipments of the same size, so that the total volume of trade between two countries is simply proportional to the number of shipments between the two countries. I consider this special case in section 3 when solving for firm level and aggregate trade only because it saves on notation, but all results are robust to relaxing it.

#### A.4 The dynamic problem of the firm

Given the static firm and general equilibrium above, I turn to the dynamic problem of the firm.

*Initial conditions.*— A firm is born with a measure  $K_0$  of suppliers and customers. Furthermore, all its suppliers and customers are also newborn firms with  $K_0$  suppliers and customers themselves. Subsequently, a firm endogenously acquires a growing number of suppliers and customers. Since all firms start with the same initial conditions, they have exactly the same incentive to acquire new suppliers and customers, so that their respective sets of suppliers and customers grow at the same rate, and the symmetry condition (20) is satisfied at all times.

*The market for information.*— I have shown above firms benefit from acquiring more suppliers and customers. I now introduce an explicit market for information about new contacts. While a priori, firm  $i$  may buy and sell information about both suppliers and customers, I assume  $i$  actively buys information about new suppliers from its existing suppliers, actively sells information about its suppliers to its customers, but passively waits to be contacted by new downstream customers. In a symmetric equilibrium, this simplification is innocuous: if  $i$  is a supplier to  $j$  then  $j$  is a customer of  $i$ . Implicitly, the justification for assuming that firm  $i$  buys information about new suppliers from its existing suppliers (and not from other firms) is that firm  $i$  has specific needs for intermediates, and  $i$ 's suppliers are in a privileged position to know about those specific needs. The sequence of contact formation is depicted in Figure 3.

I assume an exogenous price for each new name of a supplier equal to  $p_I$  units of equipped labor.<sup>20</sup> Just as  $i$  buys information at a price  $p_I w$ , it also sells information at the same price.

In addition to the direct cost  $p_I w$  of buying information, firm  $i$  faces a convex adjustment cost  $wC(I_i, K_i)$ .  $I_i$  is the “investment” into “information” about new suppliers, and  $K_i$  is the

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<sup>20</sup>A possible explanation for this pricing strategy is that firms have access to an outside technology for finding new contacts at a cost  $p_I w$  so that prices are determined by an arbitrage condition. Note also firm  $i$  always has the outside option of not purchasing information. It may happen that the benefit of one extra contact is below its price  $p_I w$ . This possibility however never occurs along the equilibrium steady state growth path. I am grateful to Volker Nocke for bringing that possibility to my attention.

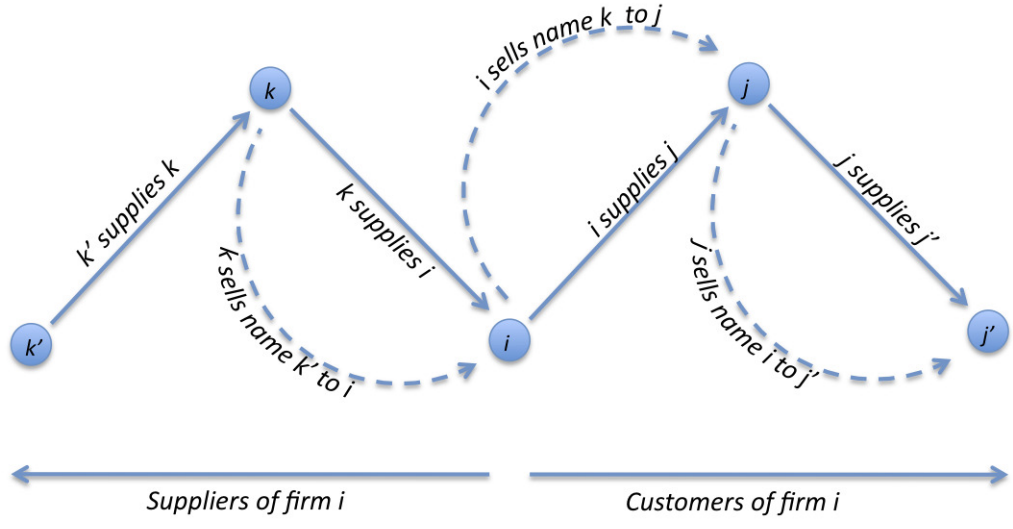


Figure 3: Firms buy and sell information about suppliers and customers.

Notes: The straight solid arrows represent input-output linkages: e.g. firm  $k$  supplies intermediate inputs to firm  $i$ . The curvy dotted lines represent information linkages: e.g. firm  $k$  sells to firm  $i$  information about a new supplier  $k'$ . After these information exchanges take place, firm  $i$  has a new supplier,  $k'$ , and a new customer,  $j'$ .

current measure of “contacts,” or suppliers of  $i$ , which plays a role analogous to “capital” in a classical production function. The adjustment cost function is assumed increasing and convex in  $I$ ,  $C_I, C_{II} > 0$ , decreasing in  $K$ ,  $C_K < 0$ , and homogenous of degree one in  $I$  and  $K$ . This convex adjustment cost function captures the idea that bringing a new supplier into a firm’s production process entails some cost: inputs have to be customized to fit into the production process, and the production process itself has to be adapted to be combined with this new source of inputs.<sup>21</sup> It is analogous to the adjustment cost assumed in the classical theory of investment, as in Lucas (1967) or Hayashi (1982). As in the investment literature, I assume that the more suppliers a firm already has ( $K_i$ ), the more efficient it is at acquiring new suppliers ( $I_i$ ), in such a way that the adjustment cost  $C$  is proportional to  $K_i$  for a given investment share  $I_i/K_i$ . As in Lucas (1967), this homogeneity assumption will warrantee that Gibrat’s law holds, in the sense that in equilibrium, the growth rate of a firm is independent of its size.

Finally, I assume that firm  $i$ ’s existing contacts are lost at some exogenous rate  $\delta$ .

<sup>21</sup>For those readers that are not satisfied with the explicit market for information I described above, it is perfectly fine to assume information is free,  $p_I = 0$ , and  $C(0, K) > 0$ . The cost of adding a single contact,  $C(0, K)$ , replaces the price of information.

The dynamic problem of the firm.— Firm  $i$  started at time  $t$  solves,

$$\max_{I_i(s)} V(t) = \int_t^{+\infty} e^{-r(s-t)} \left( K_i(s)^\alpha \frac{D_i(s)}{\sigma} - p_I w(s) I_i(s) + p_I w(s) I(s) - wC(I_i(s), K_i(s)) \right) ds \quad (25)$$

$$\text{s.t. } \dot{K}_i = I_i - \delta K_i$$

Firm  $i$  maximizes a discounted stream of profits, with a constant discount rate  $r$ . The first term represents per period profits, net of spending on intermediate inputs and labor, but before spending and receipts on information acquisition. It is a fraction  $1/\sigma$  of the aggregate sales,  $K_i(s)^\alpha D_i(s)$ , where I have separated the direct (positive) impact of the number of firm  $i$ 's suppliers on its sales,  $K_i(s)^\alpha$ , and collected all the other terms not under  $i$ 's control into a residual demand term,  $D_i(s)$ . Firm  $i$  optimally purchases information about  $I_i(s)$  new suppliers at a price  $p_I w$  each. It also sells information about  $I(s)$  suppliers at a price  $p_I w$  each, but the amount sold is beyond  $i$ 's control, as  $i$  passively waits for information buyers. Finally, firm  $i$  pays the convex adjustment cost  $wC$  to integrate new suppliers into its production process. The only decision variable for  $i$  is its “investment” into new suppliers,  $I_i(s)$ , which contributes to the stock of  $i$ 's suppliers  $K_i(s)$ .

The solution to this classical problem is such that the “investment” rate is independent of the stock of “capital” (Gibrat’s law) as in Lucas’s or Hayashi’s classical models. In other words, firm  $i$  increases its number of suppliers  $K_i$  at a rate that is independent of  $K_i$ ,

$$I_i(t) = \beta \times K_i(t)$$

where  $\beta$  is a function all the deep parameters of the model and the interest rate  $r$  that summarizes the contributions of the production function and the adjustment cost function that are relevant for the optimal investment decision. I derive formally in the next section the optimal investment policy function  $\beta$  that solves (25) and show that  $\beta$  is constant along a balanced growth path.

## A.5 Optimal investment

The firm chooses an optimal investment policy so as to maximize a discounted sum of profits as in (25), taking as given the behavior of other firms. For a generic production function and adjustment cost function, the Lagrangian associated with the firm’s program is,

$$\max_{I_t} \mathcal{L} = \int_0^{+\infty} e^{-rt} \left( \pi(K_t) - p_I w I_t - wC(I_t, K_t) + \lambda_t (\dot{K}_t - I_t - \delta K_t) \right) dt$$

where  $\pi$  is the profit function once the optimal choice of spending on labor and intermediate input has been made, and where I have omitted the proceeds from selling information, which are not affected by the firm's choices. The first order conditions for this program are,

$$\text{for all } t\text{'s, } \begin{cases} \frac{\partial \mathcal{L}}{\partial I_t} = 0 \\ \frac{\partial \mathcal{L}}{\partial K_t} = \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial K_t} \end{cases}$$

From those conditions and  $\dot{\lambda} = 0$ , I characterize the optimal investment choice,

$$(p_I w + w C_I) (\delta + r) = \pi_K - w C_K \quad (26)$$

When both the production function and the adjustment cost functions are homogenous, the solution to Equation (26) is such that the investment rate ( $I/K$ ) is constant and does not depend on the level of capital  $K$ . It is easy to see this result for the special case of a quadratic adjustment cost function, and the Cobb-Douglas production function I use in Section A, where closed form solutions can be derived. Assume the cost of bringing new suppliers into production is,

$$C(I, K) = \frac{\kappa I^2}{2 K}$$

Then the investment rate ( $I/K$ ) simply solves the quadratic equation,

$$w \left( p_I + \kappa \frac{I}{K} \right) (\delta + r) = \alpha \frac{\pi}{K} - w \frac{\kappa}{2} \left( \frac{I}{K} \right)^2 \quad (27)$$

For a given technology, the investment rate  $I/K$  is independent of a firm's size. If one firm finds it optimal to invest at this rate, in a symmetric equilibrium, all other firms will also invest at the same rate, and the conditions for  $\pi/K$  to be constant are indeed satisfied. I call the equilibrium investment rate  $\beta$ . The measure of suppliers of any firm grows at a constant rate  $\beta - \delta$ .

Because all firms are charging the same price  $p_I w$  per contact information, and because all suppliers enter the production function symmetrically, firm  $i$  has no reason to direct its search to any particular supplier  $k \in \mathcal{S}_i(t)$ . To break this indeterminacy, I assume that the  $I_i$  new names are randomly drawn (uniformly) from the set of all existing suppliers  $\mathcal{S}_i(t)$ . This means that any one of the existing suppliers  $k \in \mathcal{S}_i(t)$  reveals one of the names of its suppliers,  $k' \in \mathcal{S}_k(t)$ , with a probability  $\beta dt$  over a small time interval  $dt$ . For the same reason, to break the indeterminacy of which name  $k' \in \mathcal{S}_k(t)$  gets revealed by firm  $k$ , I simply assume that  $k$  draws  $k'$  at random among all its existing  $\mathcal{S}_k(t)$  contacts.

## A.6 Entry decision and balanced growth path

Finally, I solve for the entry choice of new firms. After entry, monopolistic firms are able to extract a surplus from their consumers. This stream of profits attracts new entrants as long as its discounted value exceeds the fixed entry cost.

In this model, as in any model where productivity growth is driven by the accumulation of one factor of production (here the diversity of suppliers,  $K$ ) combined with equipped labor under constant returns to scale, growth is ultimately driven by population growth, as in Solow (1956). I will now characterize the entry decision of firms along a balanced growth path. Along this balanced growth path, population growth induces a continuous entry of new firms.

*Free entry condition.*— Each period, the symmetric equilibrium described in section A.3 holds, and all firms “invest” in acquiring new suppliers at the constant rate  $\beta$  as described in section A.4.

Consider a potential entrant that contemplates entering at time  $t$ . Its per period profits would be a fraction  $1/\sigma$  of its sales, given in Equation (23). In a symmetric equilibrium, the firm would spend each period exactly as much in acquiring information about new suppliers as it would earn selling information about its consumers. The value of entry at time  $t$ ,  $V(t)$ , is the discounted sum of those per period profits. Firms will enter in period  $t$  as long as it exceeds the cost of entry  $wf_E$ .

The perfectly elastic supply of potential entrants implies that the value of entry must exactly equal the cost of entry in every period,

$$V(t) \equiv \int_t^{+\infty} e^{-r(s-t)} \left( \frac{K(s)^{\frac{\alpha}{1-\alpha}} wL(s)/P(s)^{1-\sigma}}{\sigma - \alpha(\sigma - 1)} - wC(I(s), K(s)) \right) ds = wf_E, \forall t \quad (28)$$

From those free entry conditions, I solve for the entry of firms along a balanced growth path.

*Balanced growth path equilibrium.*— After a firm enters, its measure of suppliers grows at a rate  $\beta - \delta$ , where  $\beta$  is the endogenous “investment” rate into new suppliers, and  $\delta$  is the exogenous rate at which existing suppliers are lost. The measure of suppliers at age  $s \geq 0$  is then  $K(s) = K_0 e^{(\beta - \delta)s}$ . The “investment” rate is constant, so that  $I(s) = \beta K(s)$ . The homogeneity of the adjustment cost function  $C$  implies that  $C(I(s), K(s)) = K(s) C(I(s)/K(s), 1) \equiv K(s) c(\beta)$ .

I assumed a constant population growth rate  $\gamma$  so that  $L(s) = L_0 e^{\gamma L s}$ . Along a balanced growth path, the measure of firms grows at some constant endogenous rate  $\gamma$ . The ideal price index given in Equation (22) is therefore such that  $P(s)^{1-\sigma} = M_0 \mathbb{E}_F \left[ K^{\frac{\alpha}{1-\alpha}} \right] e^{\gamma s}$ .

Combining those with the free entry condition (28), the following condition holds each period,

$$\int_t^\infty e^{-r(s-t)} \left( \frac{K_0^{\frac{\alpha}{1-\alpha}} e^{\frac{\alpha}{1-\alpha}(\beta-\delta)(s-t)} w e^{\gamma L s}}{(\sigma - \alpha(\sigma - 1)) M_0 \mathbb{E} \left[ K^{\frac{\alpha}{1-\alpha}} \right] e^{\gamma s}} - K_0 e^{(\beta-\delta)(s-t)} c(\beta) \right) ds = w f_E, \forall t \quad (29)$$

For this equation to hold for all  $t$ ,  $\gamma = \gamma_L$  is necessary.<sup>22</sup> In other words, the measure of firms in every location grows at the same constant rate  $\gamma$  as the population. Once this condition is imposed, Equation (29) pins down  $M_0$  and the measure of firms each period. The existence and uniqueness of a balanced growth path equilibrium is warranted.

Here is a recap of the conclusions from the above model. Firms are continuously born at a rate  $\gamma$ . A firm is born with an initial mass  $K_0$  of contacts (both suppliers and customers). Subsequently, contacts are randomly created at a rate  $\beta$  and lost at a rate  $\delta$ , with each new contact coming from the contacts of the firm's existing contacts.  $\gamma$  and  $\beta$  are endogenously determined, but constant along a balanced growth path equilibrium;  $\delta$  is exogenous. In addition, and under the simplifying assumption that the share of intermediate inputs equals that of equipped labor, a firm's size is proportional to its number of contacts, and shipment sizes are the same for all firms.

Section 3 of the main paper takes directly  $(\gamma, \beta, \delta, K_0)$  as parameters, embeds this model into geographic space and characterizes explicitly firm level trade flows, i.e., trade between the suppliers and customers of this model.

## B Welfare analysis: static and dynamic costs of trade disruptions

In this model, strictly speaking, there are *in the long run* no welfare or efficiency gains from international trade integration, nor any welfare or efficiency losses from trade disruptions. There are however potentially large welfare costs from trade disruptions *in the short run*. I will first briefly discuss the long run properties of this model, and then analyze the short and medium run response to trade shocks.

### B.1 The long run (non-existent) gains from trade

The prediction of no gains from trade in the long run is very stark, and comes from the specific assumptions of the model. It is however worth discussing briefly.

<sup>22</sup>The free entry condition is of the form,  $\frac{e^{\gamma L t}}{e^{\gamma t}}$  constant – constant = constant. It holds for any  $t$  only if  $\gamma = \gamma_L$ .

My model is a direct extension of Krugman (1980), but its predictions regarding the long run gains from trade differ starkly from the classical Krugman model. While in the Krugman model, in the absence of trade frictions, consumers would purchase all goods produced worldwide, in my model, even in the absence of any trade frictions, each firm would only purchase a (potentially small) subset of all intermediate goods available worldwide. This is due to the adjustment cost function that prevents firms from using too many intermediates at any point in time. The constraint on the use of new intermediate inputs is not external (trade friction prevent firms from using foreign intermediates), but internal (firms choose not to use all intermediates because of adjustment costs). In that sense, if a large enough country<sup>23</sup> were to move to autarky, the efficiency of any one firm, and hence aggregate efficiency, would not be affected in the long run.

This is of course an extreme prediction. It is likely part of the adjustment cost of bringing new suppliers into a firm’s production process corresponds to finding the right supplier. There may not be an infinite number of such suppliers in every location of every country, as I assume. But more constructively, this prediction only holds in the long run. In the short and medium run, disrupting existing trade linkages can potentially entail large aggregate welfare and efficiency costs. I now turn to the short run prediction of my model.

## **B.2 The short and medium run cost from trade disruption**

This model features complex production chains that cross national boundaries many times. A disruption to the international flow of goods would affect those production chains adversely. Moreover, since the process of acquiring upstream suppliers and downstream customers is a timely and costly process, rebuilding the affected segments of those production chains would take time. I consider now a simple example of such a disruption to the international flow of goods, and analyze its impact on aggregate welfare, both in the short and in the medium run.

The full model I build in sections **A** and **3** is complex, and ultimately, my ability to derive analytical solutions to the model rely on some strong simplifying assumptions. The most important assumption is that of symmetry: a firm of age  $t$  not only is connected only to other firms of age  $t$ , but those other firms have the same geographic distribution of contacts, where only coordinates are shifted. A generic disruption to the flow of goods would break this symmetry assumption: for instance, if say a large country (a long interval of the real line) were to move to complete autarky,

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<sup>23</sup>Formally, a country is “large” in my model if it contains a positive measure of locations. A “large” country can therefore be arbitrarily small compared to the size of the world.

then firms that are near the international border of that country (near the bounds of the interval) would be more severely affected than firms in the center of that country (in the middle of the interval). Since production chains are infinite, different segments of those chains would be affected differently by such a move to autarky. While this is an interesting example to consider, the added complexity would force me to make a series of ad hoc statements on all parameters of the model. I consider instead a simpler example of trade disruption. Most of the intuition for a more complex type of disruption is contained in that example, but the analysis remains simple.

*A simple example of trade disruption.*— Consider the following unanticipated shock: at a given point in time, any contact between two firms at a distance from each other greater than  $\bar{x}$  is severed. This shock is similar to a country of size  $2\bar{x}$  moving to complete autarky, except that it preserves the geographic symmetry of the system. The lower  $\bar{x}$  is, the more trade is disrupted.

Because older and larger firms are more likely to export over long distances, those firms are more likely to lose some of their contacts. This direct effect reduces trade. In addition, with the loss of contacts, the productivity of a firm goes down, so that it produces lower quantities, which further reduces trade. This affects not only the firm directly, but with each of the supplier of the firm losing suppliers themselves, this productivity effect is amplified along the entire production chain. As for the direct effect, older and larger firms are connected to more complex production chains that span wider geographic ranges, so that production and trade along those more complex production chains is more severely affected by trade disruption.

I now characterize all those effects explicitly.

*Aggregate welfare before the shock.*— Before the shock, in the symmetric equilibrium I consider in section A, a firm with  $K$  suppliers only buys from other firms with  $K$  suppliers, so that the price each firm along those  $K$ -type production chains is explicitly solved for in Equation (21). Along a balanced growth path equilibrium, the distribution of the number of contacts across firms is invariant, and only the overall mass of firms grows. All workers worldwide have the same wage, and their per period utility  $U$  depends on this wage  $w$  and the local ideal price index  $P$  they face,  $U = w/P$ . Aggregate welfare before the shock is,

$$U_{\text{before}} = w/P = w \left( \int p_{\omega}^{1-\sigma} d\omega \right)^{\frac{1}{\sigma-1}} = w \left( M \int_0^{\infty} \gamma e^{-\gamma t} K_t^{\frac{\alpha}{1-\alpha}} dt \right)^{\frac{1}{\sigma-1}} \quad (30)$$

Note that larger firms (larger  $K$ 's) contribute more to aggregate welfare, as in any model with heterogeneous firms such as Melitz (2003). Those firms also export further away on average, so they will be harder hit by any trade disruption.

*Aggregate welfare after the shock.*— After the shock, all trade linkages over a distance  $\bar{x}$  are severed. Because of the presence of adjustment costs, firms that lose their distant contacts will not instantaneously rebuild those contacts. So upon impact, a fraction of the suppliers of all firms are simply lost. Losing those suppliers induces a productivity loss, and ultimately, welfare drops in every location. Aggregate welfare after the shock is,

$$\begin{aligned} U_{\text{after}} &= w \left( M \int_0^\infty \gamma e^{-\gamma t} \left( \int_0^{\bar{x}} k_t(x) dx \right)^{\frac{\alpha}{1-\alpha}} dt \right)^{\frac{1}{\sigma-1}} \\ &= w \left( M \int_0^\infty \gamma e^{-\gamma t} \left( K_t \left( 1 - e^{-\bar{x}/\sqrt{\Delta_t}} \right) \right)^{\frac{\alpha}{1-\alpha}} dt \right)^{\frac{1}{\sigma-1}} \end{aligned} \quad (31)$$

Upon impact of the shock, the measure of firms  $M$  does not change, as firms have already paid the sunk entry cost. Because it is normalized, the nominal wage  $w$  does not change either. The only variable that adjusts is the efficiency of various firms, as they lose some suppliers from the disruption of trade linkages.

A firm with  $K_t$  suppliers has suppliers distributed over space according to the density  $k_t(x)$ . After the shock hits, this geographic distribution of suppliers is truncated above  $\bar{x}$  as in the first equality of Equation (31). Larger firms have more dispersed suppliers, a larger fraction of their suppliers are at a distance beyond  $\bar{x}$ , and they lose a larger fraction of their suppliers. This can be seen explicitly in the second equality of Equation (31), where I use the special closed-form case  $f_0 = k_0/K_0 \sim \text{Laplace}$  from proposition 2. A firm with  $K_t$  suppliers has contacts that are at an average squared distance  $\Delta_t$  away. The more suppliers a firm has, the larger  $K_t$ , the larger is  $\Delta_t$ , and the smaller is the fraction  $\left( 1 - e^{-\bar{x}/\sqrt{\Delta_t}} \right)$  of suppliers that are at a distance lower than  $\bar{x}$ . Larger firms are therefore more affected by trade disruption than smaller ones. Obviously, because of the roundabout nature of production chains, the higher the share of intermediate inputs in production,  $\alpha$ , the larger the welfare cost of trade disruptions, as the effect of the disruption of input-output linkages for one firm cascading down to its downstream suppliers gets magnified.

*Transitional dynamics.*— After the initial shock, new firms enter, and existing firms gradually rebuild linkages with new suppliers, under the constraint that no supplier can be at a distance larger than  $\bar{x}$ . Both margins, the entry of new firms, and the growth of existing firms, work together to bring back the economy to a balanced growth path equilibrium.

Analyzing the free entry condition in Equation (28), the drop in the ideal price index due to the disruption of trade linkages makes entry more profitable, and induces an influx of new firms

at a rate above its steady state. Analyzing the optimal investment equation in Equations (26) or (27), the fall in the price index increases firm level profits for all firms ( $\pi$ ), and the loss of existing contacts ( $K$ ) increases further the level of profits per supplier ( $\pi/K$ ) for all firms. Both forces increase the investment rate for all firms above its steady state level. Moreover, while profits increase by the same proportional amount for all firms, larger firms (higher  $K$  firms) lose disproportionately more suppliers, so that the profits per supplier increase more for larger firms, and those firms invest into acquiring new suppliers at a faster rate than smaller firms.

All those forces together bring back the economy towards its balanced growth path equilibrium: new firms enter at a higher rate, existing firms invest into finding new suppliers at a higher rate, and larger firms invest at a higher rate than smaller ones. During the transition, and compared to the balanced growth path equilibrium, there are too many small firms, and too few large ones. Because of the convexity of the adjustment cost function, firm growth cannot restore the economy to its balanced growth path instantaneously, and initially, the entry margin is the most active.

On a more intuitive level, right after the shock hits the world economy, there are “too many” workers chasing “too few” firms. The entry of new firms and the growth of existing ones restores the balance between the size of the labor force and the diversity of firms.

This analysis of the dynamic response to a trade disruption is but a simple example. It shows how the model developed in this paper can be used to study several aspects of the dynamic response to various shocks of an economy characterized by complex vertical production chains. In the next section, I empirically test some of the main predictions of the model.

## C Additional empirical evidence

In this section, I explore the robustness of the tests of proposition 1 presented in section 2.

### C.1 Robustness to alternative measures

I present additional statistics on the test of condition (iii) and proposition 1, and replicate the results in table 1 using a value-weighted definition of the average squared distance of exports. Formally, instead of using equation (3), I define  $\Delta(K_b)$  as

$$\Delta(K_b) = \sum_c (Distance_{France,c})^2 \left( \frac{\sum_{i \in b} exports_{i,c} \mathbf{1}[exports_{i,c} > 0]}{\sum_c \sum_{i \in b} exports_{i,c} \mathbf{1}[exports_{i,c} > 0]} \right). \quad (32)$$

Table 2 and figure 4 simply replicate the results in table 1 and figure 7 using the value-weighted definition of  $\mu$  in equation (32) instead of the unweighted definition of equation (3). While the

Table 2: Testing proposition 1 across all firms: Robustness.

<i>Condition (i):</i>	
Distribution of firm sizes:	$\lambda = 1.0048$ ( <i>s.e.</i> = 0.0213, $R^2 = 0.981$ )
<i>Condition (ii):</i>	
Average squared distance of exports:	
Unweighted average, eq. (3):	$\mu = 0.113$ ( <i>s.e.</i> = 0.0078, $R^2 = 0.817$ )
Value-weighted average, eq. (32):	$\mu = 0.062$ ( <i>s.e.</i> = 0.015, $R^2 = 0.244$ )
<i>Condition (iii):</i>	
Parameter restriction ( $\lambda < 1 + \mu$ ):	
Unweighted average, eq. (3):	$\Pr(\lambda \geq 1 + \mu) = 0.02\%$
Value-weighted average, eq. (32):	$\Pr(\lambda \geq 1 + \mu) = 5.31\%$
<i>Distance elasticity of trade:</i>	
All distances:	$\zeta_{all} = 0.767$ ( <i>s.e.</i> = 0.111, $R^2 = 0.810$ )
Long distances ( $> 2,000km$ ):	$\zeta_{long} = 1.090$ ( <i>s.e.</i> = 0.215, $R^2 = 0.720$ )
Predicted distance elasticity of trade:	
Unweighted average, eq. (3):	$1 + 2 \frac{\lambda-1}{\mu} = 1.086$ ( <i>s.e.</i> = 0.520)
Value-weighted average, eq. (32):	$1 + 2 \frac{\lambda-1}{\mu} = 1.156$ ( <i>s.e.</i> = 1.737)
<i>Proposition 1:</i>	
Wald test for $\zeta_{long} = 1 + 2 \frac{\lambda-1}{\mu}$ , eq. (3):	p-value of $\chi^2$ test = 98.3%
Wald test for $\zeta_{long} = 1 + 2 \frac{\lambda-1}{\mu}$ , eq. (32):	p-value of $\chi^2$ test = 99.4%

*Notes:* This table reproduces the tests in table 1, using both the unweighted definition of the average squared distance of exports in equation (3), and the value-weighted definition in equation (32).

estimates for  $\mu$  under a value-weighted definition of the average squared distance of exports are more dispersed than under an unweighted definition, the results presented in section 2 are robust.

## C.2 Relaxing the conditions in proposition 1

Three things can go wrong when bringing proposition 1 to the data:

1. The distribution of firm sizes is close to but not exactly a Pareto distribution.
2. The average squared distance of exports is close to but not exactly a power function of size.
3. Proposition 1 is an asymptotic result. It holds exactly only for distances going towards  $+\infty$ , whereas distances in the real world are finite.

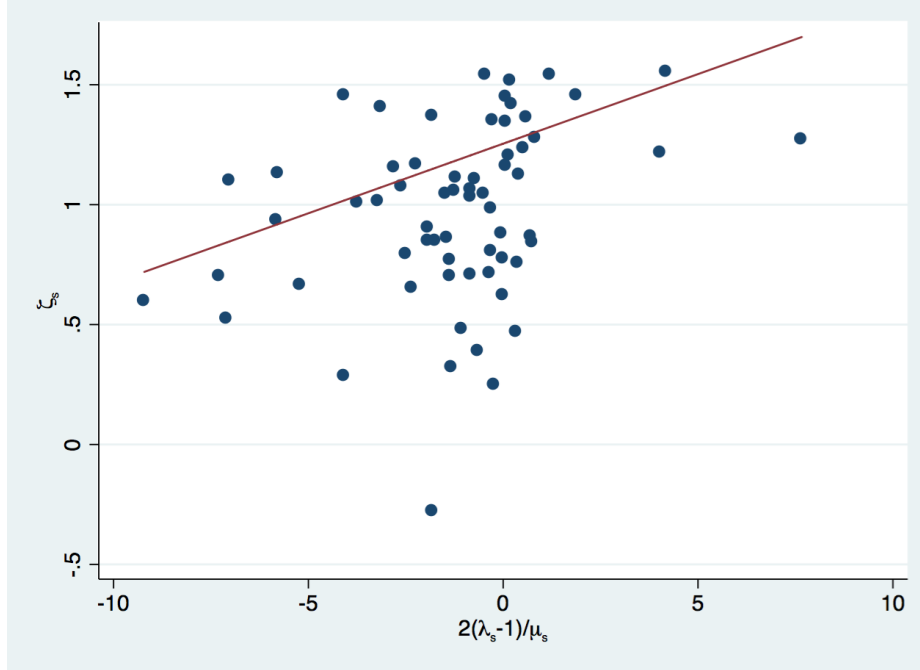


Figure 4: Proposition 1 across sectors: Robustness.

*Notes:* This figure reproduces figure 2, using the value-weighted definition of the average squared distance of exports in equation (32) instead of the unweighted definition of equation (3). The results from a GLS regression of equation (7) are:  $\hat{\alpha} = 1.254$  (*s.e.* = 0.0121),  $\hat{\beta} = 0.058$  (*s.e.* = 0.0055), and the  $R^2$  is 64%. The raw correlation between  $\zeta_s$  and  $2(\lambda_s - 1)/\mu_s$  is 31%, and the correlation where the  $2(\lambda_s - 1)/\mu_s$ 's are weighted by the inverse of their bootstrapped variance is 81%.

I address all three concerns together. To do so, I simulate the exports of synthetic firms. Using data on French exporters, I calibrate exactly the actual distribution of firm sizes (which is not exactly Pareto), and the average squared distance of exports (which is not exactly a power function of firm size), using the actual distances between countries (which are all finite). My simulated data match exactly both the distribution of exporters sizes and the average squared distance of exports from the data as in figure 1. I choose a particular function to govern firm level trade, imposing that the average squared distance of exports for firms of various sizes matches that from the data. I then compare the simulated aggregate trade flows from these synthetic firms to actual aggregate trade flows. I confirm that simply matching those two sets of moments (the firm size distribution and the average squared distance of exports as a function of firm size) is enough to replicate the geography of aggregate trade flows, both over short and long distances. I conclude that the two conditions identified in proposition 1 are quantitatively important conditions to explain the geography of aggregate trade.

*Simulation procedure.*— I take from the data all French exporters with more than 1 million French Francs of exports for each year  $t = 1986, \dots, 1992$ . To cut on the amount of moments matched, I construct 50 bins of equal log width,  $b = 1, \dots, 50$ , ranging from 1 million French Francs to the actual largest amount exported by a single firm. Firm size is the value of worldwide exports.

I directly read from the data the number of French firms in each size bin,

$$N_{b,t}^{simul} = N_{b,t}^{data} \quad (33)$$

I also read from the data the average export size in each bin,

$$K_{b,t}^{simul} = \frac{\sum_{i \in b} exports_{i,t}^{data}}{N_{b,t}^{data}} \quad (34)$$

The  $N_{b,t}^{simul}$ 's and  $K_{b,t}^{simul}$ 's discipline the distribution of firm sizes in my simulated data.

I assume the geography of trade among firms in bin  $b$  and year  $t$  follows an exponential type distribution<sup>24</sup> governed by a single parameter  $\kappa_{b,t}$ . Only the parameters  $\kappa_{b,t}$ 's will be calibrated from the data. The fraction of simulated firms in bin  $b$  and year  $t$  that export to country  $c$  is,

$$f_{b,t}^{simul}(\kappa_{b,t}, c) = \frac{\exp(-\kappa_{b,t} Distance_{France,c})}{\sum_{c'} \exp(-\kappa_{b,t} Distance_{France,c})} \quad (35)$$

where I directly read from the data the distances between France and all other countries. This fraction is the analog to the empirical fraction of firms in bin  $b$  and year  $t$  exporting to country  $c$ ,

$$f_{b,t}^{data}(c) = \frac{N_{b,t}^{data}(c)}{\sum_{c'} N_{b,t}^{data}(c')}$$

I calibrate the  $\kappa_{b,t}$ 's to match exactly the average squared distance of exports in each bin,

$$\kappa_{b,t} \text{ s.t. } \sum_c (Distance_{France,c})^2 f_{b,t}^{simul}(\kappa_{b,t}, c) = \sum_c (Distance_{France,c})^2 f_{b,t}^{data}(c) \quad (36)$$

The  $\kappa_{b,t}$ 's discipline the average squared distance of exports for firms of different sizes.

The calibrated parameters  $N_{b,t}^{simul}$ 's,  $K_{b,t}^{simul}$ 's and  $\kappa_{b,t}^{simul}$ 's are such that both panels of figure 1 would be exactly identical on simulated and actual data.

Simulated aggregate exports are the sum of exports of firms from all bins,

$$Trade_{France,c}^{simul} = \sum_b N_{b,t}^{simul} K_{b,t}^{simul} f_{b,t}^{simul}(c) \quad (37)$$

I can now estimate various specifications of the gravity equation on simulated and actual aggregate trade flows, and assess the robustness of proposition 1 in matching actual trade patterns.

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<sup>24</sup>Proposition 2 offers a theoretical justification for the choice of the exponential function.

*Results: constant trade elasticity.*— I first estimate a basic regression of the log of trade on the log of distance. Separately for each year, I run on actual and simulated data an OLS regression with importer and exporter fixed effects,<sup>25</sup> as Head and Mayer (2014) suggest is good practice,

$$\ln \left( Trade_{A,B,t}^{data} \right) = c_{A,t} + c_{B,t} - \zeta_t^{data} \ln \left( Distance_{A,B} \right) + u_{A,B,t} \quad (38)$$

$$\ln \left( Trade_{France,B,t}^{simul} \right) = c_t - \zeta_t^{simul} \ln \left( Distance_{France,B} \right) + u_{B,t} \quad (39)$$

The results are presented in table 3. Across all years, the distance elasticity of aggregate trade is close to -1 both in the actual and simulated data. Moreover, the distance elasticities on actual and simulated data are statistically indistinguishable from each other, with p-values above 80% except in 1990 (still, 64%). The simulated data even picks up the gradual decrease of the distance elasticity over the period noticed by Disdier and Head (2008), falling from around -1 to -1.1.

These simulations suggest that the two conditions identified in proposition 1 are key to understanding the distance elasticity of aggregate trade, even when they are not exactly satisfied.

*Results: non parametric distance effect.*— To further explore the empirical fit of proposition 1 over both short and long distances, I allow for a non parametric distance effect. As has been noticed since at least Eaton and Kortum (2002), the distance elasticity in the actual data does not seem to be exactly constant. To capture this non linear effect, I estimate the gravity equation non parametrically, allowing for a flexible distance effect. For each year separately, I estimate the following OLS regression with importer and exporter fixed effects using actual and simulated data,

$$\ln \left( Trade_{A,B,t}^{data} \right) = c_{A,t} + c_{B,t} + \sum_l \zeta_{l,t}^{data} \mathbb{I} [Distance_{A,B} \in l] + u_{A,B,t} \quad (40)$$

$$\ln \left( Trade_{France,B,t}^{simul} \right) = c_t + \sum_l \zeta_{l,t}^{simul} \mathbb{I} [Distance_{France,B} \in l] + u_{B,t} \quad (41)$$

where I define 6 distance intervals (measured in km's) for  $l$ : [0, 500); [500, 1000); [1000, 3000); [3000, 5000); [5000, 10000); and [10000, max]. Those distance intervals are roughly the same as in Eaton and Kortum (2002).  $\mathbb{I}[\cdot]$  is the indicator function. The parameters  $\zeta_{l,t}^{data}, \zeta_{l,t}^{simul}$  capture in a flexible non-parametric way a potentially non constant distance elasticity of trade.

The results are presented in table 4. In both the actual and simulated data, the distance elasticity of trade is not constant, but decreases gradually with distance. The simulated data captures this non linear distance effect well. Pair-wise, across all years and all distance categories,

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<sup>25</sup>As my simulated data only contain exports from France, I obviously cannot include an importer fixed effect.

Table 3: Gravity regressions with constant distance elasticity, actual versus simulated data.

Year	Dataset	ln (Distance)	s.e.	N. Obs.	Adj. $R^2$
1992	Actual data	-1.08***	(0.038)	2077	0.81
	Simulated data	-1.09***	(0.052)	46	0.91
	Test “data = simul”	0.94			
1991	Actual data	-1.08***	(0.037)	2135	0.81
	Simulated data	-1.10***	(0.051)	47	0.91
	Test “data = simul”	0.89			
1990	Actual data	-1.10***	(0.041)	2165	0.77
	Simulated data	-1.02***	(0.049)	48	0.90
	Test “data = simul”	0.64			
1989	Actual data	-1.02***	(0.043)	1938	0.74
	Simulated data	-0.98***	(0.050)	47	0.89
	Test “data = simul”	0.84			
1988	Actual data	-1.01***	(0.042)	2006	0.76
	Simulated data	-1.00***	(0.050)	48	0.89
	Test “data = simul”	0.96			
1987	Actual data	-1.03***	(0.044)	1959	0.75
	Simulated data	-1.01***	(0.051)	47	0.89
	Test “data = simul”	0.91			
1986	Actual data	-1.00***	(0.047)	1679	0.73
	Simulated data	-0.98***	(0.054)	44	0.89
	Test “data = simul”	0.91			

*Notes:* This table shows the result of the OLS estimation of equations (38) on actual data and (39) on simulated data. The dependent variable is the log of aggregate exports, and the only coefficient reported,  $\zeta_t$ , is on the log of distance. Test “data = simul” gives the p-value of a formal statistical test that the coefficients  $\zeta_t^{data}$  and  $\zeta_t^{simul}$  are equal. Robust standard errors are in parentheses.

\*\*\* Significant at the 1 percent level.

\*\* Significant at the 5 percent level

\* Significant at the 10 percent level

Table 4: Gravity regressions with non parametric distance effect, actual versus simulated data.

Year	Dataset	500-999km		1,000-2,999km		3,000-4,999km		5,000-9,999km		10,000km-max		N. Obs.	Adj. $R^2$
		coeff.	s.e.	coeff.	s.e.	coeff.	s.e.	coeff.	s.e.	coeff.	s.e.		
1992	Data	0.49*	(0.285)	-0.28	(0.270)	-1.19***	(0.282)	-1.99***	(0.270)	-2.63***	(0.275)	2077	0.81
	Simulated	-0.10	(0.375)	-0.35	(0.365)	-1.08***	(0.392)	-2.25***	(0.363)	-3.22***	(0.372)	46	0.92
	Test "data = simul"	0.64		0.96		0.94		0.83		0.64			
1991	Data	0.40	(0.285)	-0.30	(0.270)	-1.11***	(0.281)	-2.01***	(0.270)	-2.64***	(0.275)	2135	0.81
	Simulated	-0.10	(0.370)	-0.35	(0.360)	-1.10***	(0.387)	-2.28***	(0.357)	-3.25***	(0.367)	47	0.92
	Test "data = simul"	0.69		0.97		0.99		0.83		0.63			
1990	Data	0.17	(0.319)	-0.82***	(0.301)	-1.80***	(0.313)	-2.43***	(0.303)	-3.12***	(0.308)	2165	0.76
	Simulated	-0.09	(0.335)	-0.33	(0.321)	-0.98***	(0.340)	-2.07***	(0.320)	-3.03***	(0.331)	48	0.92
	Test "data = simul"	0.85		0.71		0.56		0.78		0.95			
1989	Data	0.03	(0.327)	-0.90***	(0.308)	-1.79***	(0.322)	-2.42***	(0.310)	-3.02***	(0.315)	1938	0.74
	Simulated	-0.09	(0.334)	-0.32	(0.320)	-0.94***	(0.338)	-1.98***	(0.319)	-2.95***	(0.330)	47	0.92
	Test "data = simul"	0.93		0.66		0.55		0.74		0.96			
1988	Data	0.44	(0.334)	-0.46	(0.317)	-1.36***	(0.326)	-1.97***	(0.318)	-2.56***	(0.322)	2006	0.76
	Simulated	-0.09	(0.330)	-0.33	(0.317)	-0.93***	(0.330)	-2.02***	(0.316)	-2.99***	(0.327)	48	0.92
	Test "data = simul"	0.70		0.92		0.75		0.97		0.75			
1987	Data	0.49	(0.340)	-0.40	(0.323)	-1.32***	(0.333)	-1.94***	(0.325)	-2.53***	(0.329)	1959	0.74
	Simulated	-0.09	(0.335)	-0.33	(0.322)	-0.93***	(0.335)	-2.04***	(0.321)	-3.01***	(0.332)	47	0.92
	Test "data = simul"	0.68		0.96		0.78		0.94		0.73			
1986	Data	0.40	(0.346)	-0.56*	(0.327)	-1.38***	(0.339)	-1.98***	(0.329)	-2.59***	(0.336)	1679	0.73
	Simulated	-0.09	(0.341)	-0.30	(0.329)	-0.91**	(0.341)	-1.97***	(0.328)	-2.99***	(0.341)	44	0.91
	Test "data = simul"	0.73		0.85		0.73		0.99		0.77			

Notes: This table shows the result of the OLS estimation of equations (40) on actual data and (41) on simulated data. The dependent variable is the log of aggregate exports, and the only coefficients reported, the  $\zeta_{i,t}$ 's, are those for the distance intervals (measured in km's),  $l$  in [0, 500); [500, 1000); [1000, 3000); [3000, 5000); [5000, 10000); or [10000, 20000]. The omitted category is [0, 500). Test "data = simul" gives the p-value of a formal statistical test that the coefficients  $\zeta_{i,t}^{data}$  and  $\zeta_{i,t}^{simul}$  are equal. Robust standard errors are in parentheses.

\*\*\* Significant at the 1 percent level.

\*\* Significant at the 5 percent level

\* Significant at the 10 percent level

the estimated coefficients on actual and simulated data are statistically indistinguishable from each other, with all p-values above 55%, and above 75% in the majority of cases.

This non parametric estimation suggests again that the conditions identified in proposition 1 are useful for describing the patterns of aggregate trade flows, not only over long distances, as proposition 1 formally shows, but also over short distances.

## D Additional mathematical proofs

In this section, I show that even if firms are allowed to form contact with firms in other cohorts, the average squared distance of exports is still a power function of firm size, so that the main result of the paper (the gravity equation in the aggregate) still holds.

For proposition 1 to hold, it is enough that the distribution of firm sizes is Pareto and that the average squared distance of a firm's exports is a power function of its size. Given that firms grow at a constant rate, this second condition is equivalent to saying that the average squared distance of a firm's exports grows exponentially with the firm's age.

In the theoretical model of section 3, I impose firms only form contacts with firms of the same age. Under this simplifying assumption, both conditions (i) and (ii) are satisfied. When firms are allowed to form contacts with firms of different ages, the distribution of firm sizes is unaffected and condition (i) still holds; but the geography of firms trade, and the average squared distance of a firm's exports on the other hand are affected. The following proposition shows that even if firms do form contacts with firms of different ages, the average squared distance of a firm's exports still grows exponentially with age (at least asymptotically), so that condition (ii) still holds.

**Proposition 4** *If newborn firms form contacts with any existing set of firms that are on average at a finite squared distance from their own contacts, then the average squared distance from a firm's contacts,  $\Delta_t$ , grows at an exponential rate  $\lambda_t\beta$  where  $\lambda_t$  converges to a constant.*

**Proof.** Denote by  $F_t$  such that  $\int_{\mathbb{R}} F_t = 1$  the geographic distribution (density) of the contacts of a firms of age  $t$ . Under the simplifying assumption that newborn firms only get connected to other newborn firms,  $F_t = f_t = k_t/K_t$ , which renders the partial differential equation (8) for the dynamic evolution of a firm's contacts quite tractable. I now want to relax this assumption, and allow not only  $F_t \neq f_t$  but also to allow  $F_t$  to evolve endogenously itself.

First, conjecture that  $F_t$  exists and that  $Var [F_t] \equiv \Gamma_t < +\infty$ . I will confirm this conjecture below. The PDE (8) becomes,

$$\frac{\partial}{\partial t} k_t = \beta k_t * F_t - \delta k_t$$

where  $*$  denotes the convolution product. Given that the number of contacts evolves according to  $\frac{\partial}{\partial t} K_t = (\beta - \delta) K_t$ , I get the following PDE for the evolution of the normalized distribution of a firm's contacts,  $f_t \equiv \frac{k_t}{K_t}$ ,

$$\frac{\partial}{\partial t} f_t = \frac{\partial}{\partial t} \left( e^{(\delta-\beta)t} k_t \right) = e^{(\delta-\beta)t} \beta (k_t * F_t - k_t)$$

Taking the Fourier transform of the above PDE, I get,

$$\frac{\partial}{\partial t} \hat{f}_t = e^{(\delta-\beta)t} \beta \hat{k}_t (\hat{F}_t - 1)$$

Using the property that the variance of a random variable is equal to the second derivative of the Fourier transform of its p.d.f. evaluated at zero, I get,

$$\frac{\partial}{\partial t} \Delta_t = \frac{\partial}{\partial t} \hat{f}_t''(0) = e^{(\delta-\beta)t} \beta \frac{\partial^2}{\partial s^2} \left\{ \hat{k}_t(s) (\hat{F}_t(s) - 1) \right\}_{s=0}$$

I then calculate the last term of this expression,

$$\begin{aligned} h(s) &= \hat{k}_t(s) (\hat{F}_t(s) - 1) \\ h'(s) &= \hat{k}_t'(s) \hat{F}_t(s) + \hat{k}_t(s) \hat{F}_t'(s) - \hat{k}_t'(s) \\ h''(s) &= \hat{k}_t''(s) \hat{F}_t(s) + 2\hat{k}_t'(s) \hat{F}_t'(s) + \hat{k}_t(s) \hat{F}_t''(s) - \hat{k}_t''(s) \\ h''(0) &= \hat{k}_t''(0) \times 1 + 2 \times 0 + \hat{k}_t(0) \hat{F}_t''(0) - \hat{k}_t''(0) = e^{(\beta-\delta)t} \Gamma_t \end{aligned}$$

where I use the symmetry of the functions  $f_t$  and  $F_t$  to equate the derivatives of odd order of their Fourier transforms to zero,  $\hat{F}_t(0) = 1$  and  $\hat{f}_t(0) = K_t = e^{(\beta-\delta)t}$  to get the last two equalities. Plugging this expression back into the PDE for  $\Delta_t$ , I get,

$$\frac{\partial}{\partial t} \Delta_t = \beta \Gamma_t$$

I now need to characterize the evolution of  $\Gamma_t = \hat{F}_t''(0)$ . Denoting by  $w_t(t+s)$  the weight of firms of age  $t+s$  that a firm of age  $t$  is connected to,  $F_t$  is defined as follows,

$$F_t = \frac{\int_{\mathbb{R}^+} w_t(t+s) k_{t+s} ds}{\int_{\mathbb{R}^+} w_t(t+s) K_{t+s} ds}$$

I conjecture that  $\Delta_t/\Gamma_t = \lambda$  so that  $\Delta_t = \Delta_0 e^{\beta\lambda t}$ . I will confirm this conjecture below. I can now calculate  $\Gamma_t$  from the above expression for  $F_t$ ,

$$\begin{aligned}\Gamma_t = \hat{F}_t''(0) &= \frac{\int_{\mathbb{R}^+} w_t(t+s) \hat{k}_{t+s}''(0) ds}{\int_{\mathbb{R}^+} w_t(t+s) K_{t+s} ds} \\ &= \frac{\int_{\mathbb{R}^+} w_t(t+s) K_{t+s} \Delta_0 e^{\beta\lambda(t+s)} ds}{\int_{\mathbb{R}^+} w_t(t+s) K_{t+s} ds} \\ &= \frac{\int_{\mathbb{R}^+} w_t(t+s) K_{t+s} e^{\beta\lambda s} ds}{\int_{\mathbb{R}^+} w_t(t+s) K_{t+s} ds} \Delta_0 e^{\beta\lambda t} \\ &= A_t \Delta_t\end{aligned}$$

with  $A_t = \frac{\int_{\mathbb{R}^+} w_t(t+s) K_{t+s} e^{\beta\lambda s} ds}{\int_{\mathbb{R}^+} w_t(t+s) K_{t+s} ds}$ . Under some condition,  $A_t$  is either equal to a constant (which I call  $\lambda$ ), or converges to a constant (which I call  $\lambda$ ). For instance, if  $w_t(t+s) = w(s), \forall t$ , then  $A_t$  is constant. Alternatively, if  $w_t(t+s)$  is of the form  $e^{-\omega_t s}$  then  $A_t = \frac{\omega_t - (\beta - \delta)}{\omega_t - (\beta(1+\lambda) - \delta)}$ . If either  $\omega_t \rightarrow +\infty$  or  $\omega_t \rightarrow \omega > \beta(1+\lambda) - \delta$ , then  $A_t$  converges to a constant. In either case, the above conjectures  $\Gamma_t < +\infty$  for all  $t$ 's, and  $\Delta_t/\Gamma_t = \lambda$  are verified, either for all  $t$ 's, or asymptotically. ■

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