The Gravity Equation in International Trade: A Response

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This short note addresses the concerns raised by Dewitte (2022) on the empirical exercise in Chaney (2018). I gratefully acknowledge that all coding inconsistencies noted by Dewitte (2022) are to be corrected, and I show that after correcting those inconsistencies, the statistical tests in Chaney (2018) are weaker but remain statistically significant.

I. Introduction

Dewitte (2022) carefully audits the code accompanying my article (Chaney 2018) and finds several inconsistencies between the text and the code. All inconsistencies are correctly identified. I am immensely grateful for Ruben Dewitte's patience and hard work in uncovering and correcting those inconsistencies.

As shown in Dewitte (2022), correcting those inconsistencies changes several point estimates. Taking into account standard errors, I show that the main statistical tests of Chaney (2018) are weaker than those in the original version but remain statistically significant at conventional levels.

More constructively, Dewitte (2022) shows that estimating the tail index of a Pareto distribution of firm sizes is sensitive to sample selection.

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This is a well-known fact. For instance, in a seminal early contribution, Axtell (2001, 1819) notes, “There are too few very small and very large firms with respect to the Zipf fit.” This departure from a Pareto distribution for small and large firms, which I confirm holds for French exporters, implies that the tail index of a Pareto distribution of firm sizes estimated on data truncated above a lower size threshold will systematically go up (in absolute value) as the threshold increases. As a consequence, the sufficient conditions for gravity identified in Chaney (2018) hold for some subsamples (low threshold–large sample), but not for others (high threshold–small sample). I show that when the conditions hold, the prediction for gravity holds; when they are violated, the prediction does not hold. This suggests that the conditions identified in Chaney (2018) as sufficient may also be, quantitatively, necessary.

II. Replication of Chaney (2018)

Proposition 1 in Chaney (2018) identifies three sufficient conditions under which the distance elasticity of aggregate trade is asymptotically constant and, under further conditions, close to -1.

Proposition 1. If the following three conditions hold,

(i) firms sizes follow a Pareto distribution over $[K_{\min}, +\infty)$ with shape parameter $\lambda \geq 1$;

(ii) the average squared distance of exports is an increasing power function of firm size, meaning that the fraction of exports shipped at a distance x by firms of size K is given by a function $f_K(x)$ such that

$$\int_0^{\infty} x^2 f_K(x)\, dx = K^{\mu}\left(\int_0^{\infty} x^2 f_{K_{\min}}(x)\, dx\right) \quad \text{with } \mu > 0,$$

where I further impose that $f_K(x)$ and $f_K'(x)$ are bounded from above and that $f_K(x)$ is weakly decreasing above some threshold $\bar{x}$; and

(iii) $\lambda < 1 + \mu$,

then $-\zeta$, the elasticity of aggregate trade between two countries A and B normalized by country size ($\text{Trade}_{\text{A,B}}$) with respect to distance is asymptotically constant,

$$\text{Trade}_{\text{A,B}}\left(\text{Distance}_{\text{A,B}} = x\right) \underset{x \to +\infty}{\propto} \frac{1}{x^{\zeta}} \quad \text{with } \zeta = 1 + 2(\lambda - 1)/\mu.$$

Furthermore, if the distribution of firm sizes is close to Zipf’s law ($\lambda \approx 1$), then aggregate trade is inversely proportional to distance ($\zeta \approx 1$).

The empirical section in Chaney (2018; sec. III) does not formally test proposition 1. This proposition is an extremely stylized representation of reality, and taking it literally would be imprudent. Instead, Chaney (2018) estimates the Pareto tail index λ of the distribution of firm sizes under the assumption (not tested) that firm sizes are Pareto distributed. It estimates the exponent μ of the power function of average squared distance and firm size under the assumption (not tested) that the squared distance is a power function of size. It estimates the distance elasticity ζ of aggregate trade under the assumption (not tested) that aggregate trade is a power function of distance. Only then does it test whether condition (iii) holds and whether the estimates for the actual and predicted distance elasticities of trade are statistically different from each other.

The most consequential coding inconsistency identified by Dewitte (2022) is that different samples of firms are used to estimate λ (Pareto), μ (size-distance), and ζ (gravity): the estimates for μ and ζ use firms exporting above 1 million French francs (about \$200,000), while the estimate for λ uses firms exporting above 1.91 million French francs (about \$380,000). To correct this inconsistency, one can either use all firms exporting above 1 million French francs, or all firms exporting above 1.91 million French francs.

Table 1 presents corrections for both. Panel A reproduces the original table 1 of Chaney (2018, 162), including all coding inconsistencies. Panel B corrects all inconsistencies, using the set of firms exporting above 1 million French francs. Panel C corrects all inconsistencies, using the set of firms exporting above 1.91 million French francs. For both data sets, condition (iii) remains satisfied almost always (for more than 99.98% of the estimates), and the difference between the actual and predicted distance elasticities of aggregate trade remains statistically insignificant at conventional levels (all p-values are above 0.2, well above a conventional level of 0.05, or even 0.10), although all statistical tests are weaker than those in Chaney (2018).

Figure 1 displays the distribution of firm sizes and the relation between size and distance of exports: the top panel reproduces figure 1 of Chaney (2018, 161), including all coding inconsistencies. The middle panel uses the sample of firms exporting above 1 million French francs, correcting all inconsistencies. The bottom panel uses the sample of firms exporting above 1.91 million French francs, correcting all inconsistencies. The differences across panels are small.

III. Robustness of Chaney (2018) to Sample Selection

The fact that as the sample of firms above a lower size threshold shrinks, the Pareto tail index increases (in absolute value) naturally brings the question of

TABLE 1
Replications of Table 1 of Chaney (2018)

A. Exact Replication (Including All Inconsistencies)	
Condition (i): distribution of firm sizes	$\lambda = 1.0048$ (SE = .0213, $R^2 = .981$)
Condition (ii): average squared distance of exports	$\mu = .1131$ (SE = .0078, $R^2 = .817$)
Condition (iii): parameter restriction ($\lambda < 1 + \mu$)	$\Pr(\lambda \geq 1 + \mu) = .020\%$
Distance elasticity of trade:	
All distances	$\zeta_{\text{all}} = .7672$ (SE = .1108, $R^2 = .810$)
Long distances (>2,000 km)	$\zeta_{\text{long}} = 1.0902$ (SE = .2150, $R^2 = .720$)
Predicted distance elasticity of trade	$1 + 2(\lambda - 1)/\mu = 1.0856$ (SE = .5195)
Proposition 1:	
Wald test for $\zeta_{\text{all}} = 1 + 2(\lambda - 1)/\mu$	p-value of χ^2 test = 99.306%
Wald test for $\zeta_{\text{long}} = 1 + 2(\lambda - 1)/\mu$	p-value of χ^2 test = 99.344%
B. Replication with Firms above 1M FF (Correcting All Inconsistencies)	
Condition (i): distribution of firm sizes	$\lambda = .9707$ (SE = .0289, $R^2 = .977$)
Condition (ii): average squared distance of exports	$\mu = .1131$ (SE = .0084, $R^2 = .817$)
Condition (iii): parameter restriction ($\lambda < 1 + \mu$)	$\Pr(\lambda \geq 1 + \mu) = .000\%$
Distance elasticity of trade:	
All distances	$\zeta_{\text{all}} = .7672$ (SE = .0979, $R^2 = .810$)
Long distances (>2,000 km)	$\zeta_{\text{long}} = 1.1854$ (SE = .2498, $R^2 = .716$)
Predicted distance elasticity of trade	$1 + 2(\lambda - 1)/\mu = .4824$ (SE = .5141)
Proposition 1:	
Wald test for $\zeta_{\text{all}} = 1 + 2(\lambda - 1)/\mu$	p-value of χ^2 test = 58.627%
Wald test for $\zeta_{\text{long}} = 1 + 2(\lambda - 1)/\mu$	p-value of χ^2 test = 21.868%
C. Replication with Firms above 1.91M FF (Correcting All Inconsistencies)	
Condition (i): distribution of firm sizes	$\lambda = 1.0041$ (SE = .0275, $R^2 = .981$)
Condition (ii): average squared distance of exports	$\mu = .1067$ (SE = .0086, $R^2 = .788$)
Condition (iii): parameter restriction ($\lambda < 1 + \mu$)	$\Pr(\lambda \geq 1 + \mu) = .020\%$
Distance elasticity of trade:	
All distances	$\zeta_{\text{all}} = .7615$ (SE = .0975, $R^2 = .812$)
Long distances (>2,000 km)	$\zeta_{\text{long}} = 1.1766$ (SE = .2483, $R^2 = .719$)
Predicted distance elasticity of trade	$1 + 2(\lambda - 1)/\mu = 1.0770$ (SE = .5252)
Proposition 1:	
Wald test for $\zeta_{\text{all}} = 1 + 2(\lambda - 1)/\mu$	p-value of χ^2 test = 55.468%
Wald test for $\zeta_{\text{long}} = 1 + 2(\lambda - 1)/\mu$	p-value of χ^2 test = 86.396%

Note.—This table replicates table 1 of Chaney (2018, 162). Panel A is an exact replication of that table, including all inconsistencies. Panel B corrects all inconsistencies for all firms above 1 million French francs (M FF; about $200,000). Panel C corrects all inconsistencies for all firms above 1.91M FF (about $380,000).

the robustness of proposition 1 for different samples of firms. Table 2 explores this question. It replicates exactly the analysis in table 1 of Chaney (2018, 162), for different samples of firms, correcting all coding inconsistencies.[1] Starting

[1] The p-value in the last row of panel A, 99.344%, was incorrectly rounded up to 99.4% in Chaney (2018).

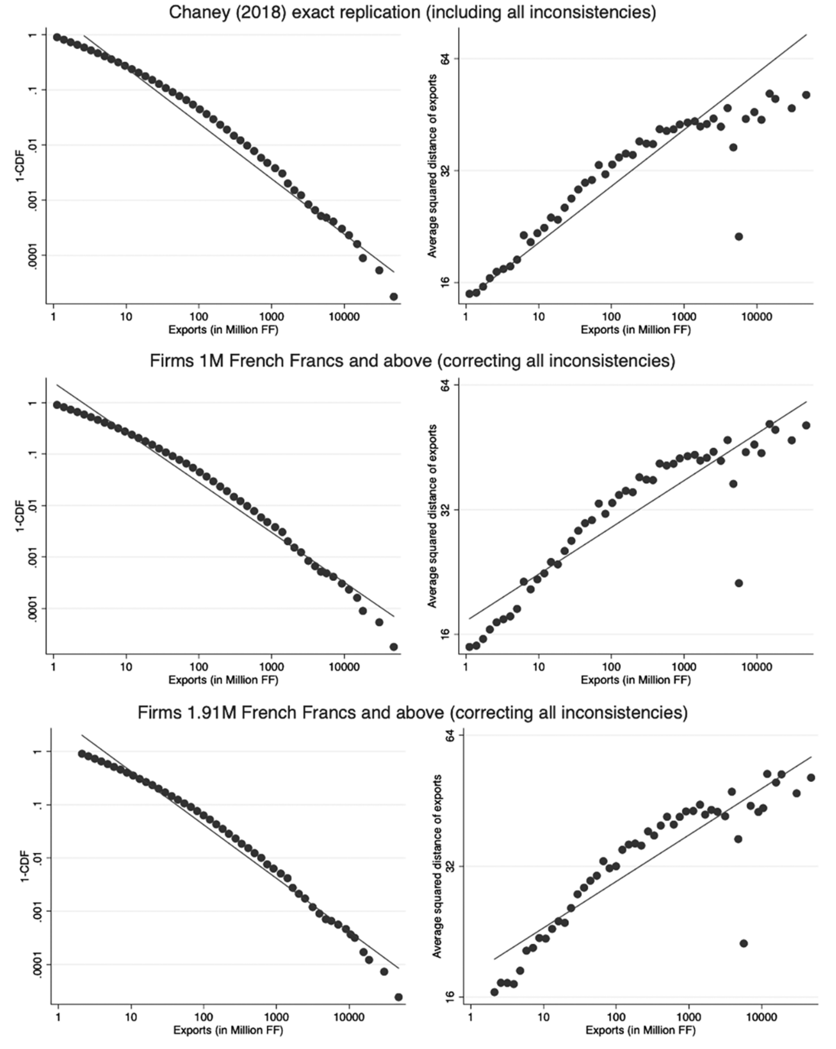

FIG. 1.—This figure replicates figure 1 of Chaney (2018, 161). The top panel is an exact replication of that figure, including all inconsistencies. The middle panel corrects all inconsistencies for all firms above 1 million French francs (FF; about \$200,000). The bottom panel corrects all inconsistencies for all firms above 1.91 million FF (about \$380,000). CDF = cumulative distribution function. A color version of this figure is available online.

with all firms exporting above 1 million French francs, I gradually increase the lower size threshold by a factor of 1.24 (above 1 million, 1.24 million, 1.54 million, 1.91 million, etc). As the sample of firms shrinks, the Pareto tail index, parameter λ in condition (i) of proposition 1, increases in absolute value. The elasticity of the average squared distance of exports to firm

TABLE 2
Replications of Table 1 of Chaney (2018) for Different Subsamples (Correcting All Inconsistencies)

Selected Firms	No. of Firms (1)	λ (2)	μ (3)	$\Pr(\lambda \geq 1+\mu)$ (4)	ζ_{all} (5)	ζ_{long} (6)	$1+2(\lambda-1)/\mu$: ζ_{theory} (7)	p-Value for test $\zeta_{all} = \zeta_{theory}$ (8)	p-Value for test $\zeta_{long} = \zeta_{theory}$ (9)
Firms above:									
1.00M FF	27,968	.971	.113	.000	.767	1.185	.482	.576	.209
		(.029)	(.008)		(.098)	(.250)	(.500)		
1.24M FF	25,301	.981	.117	.000	.766	1.183	.670	.861	.384
		(.029)	(.006)		(.098)	(.249)	(.534)		
1.54M FF	22,823	.992	.114	.000	.764	1.180	.854	.852	.542
		(.028)	(.006)		(.098)	(.249)	(.473)		
1.91M FF	20,491	1.004	.107	.000	.761	1.177	1.077	.550	.862
		(.027)	(.009)		(.097)	(.248)	(.518)		
2.37M FF	18,409	1.015	.104	.004	.760	1.175	1.298	.327	.837
		(.028)	(.009)		(.097)	(.248)	(.540)		
2.94M FF	16,445	1.026	.102	.004	.758	1.173	1.511	.174	.573
		(.026)	(.009)		(.097)	(.248)	(.546)		
3.64M FF	14,635	1.038	.105	.014	.755	1.170	1.729	.068	.335
		(.027)	(.006)		(.097)	(.247)	(.524)		
4.52M FF	12,963	1.045	.094	.044	.752	1.165	1.952	.042	.213
		(.027)	(.009)		(.097)	(.246)	(.583)		
5.61M FF	11,431	1.056	.092	.103	.748	1.161	2.221	.016	.103
		(.026)	(.009)		(.096)	(.245)	(.603)		
6.96M FF	10,100	1.068	.090	.203	.745	1.156	2.508	.008	.053
		(.027)	(.011)		(.096)	(.244)	(.653)		

Notes.—This table replicates table 1 of Chaney (2018, 162) for 10 different subsamples, correcting all coding inconsistencies identified by Dewitte (2022). Each row corresponds to a different sample of firms, keeping only firms above a lower size threshold, where I increase the threshold by a factor of 1.24 from one row to the next, starting at 1 million French francs (M FF; e.g., all firms exporting above 1M FF in the first row, above 1.24M FF in the second row, and so on). Column 1 shows the number of firms for each subsample. In col. 2, λ is the OLS (ordinary least squares) estimate of the Pareto shape parameter of the distribution of firm sizes, from eq. (4) of Chaney (2018, 160). In col. 3, μ is the OLS estimate of the size elasticity of the average squared distance of exports, from eq. (5) on page 160. Column 4 computes the fraction of bootstrapped estimates of λ and μ such that condition (iii) is satisfied, across 999 bootstraps. In col. 5, ζ_{all} is the OLS estimate of the distance elasticity of aggregate trade for all distances, from eq. (6) on page 160. In col. 6, ζ_{long} is the OLS estimate of the distance elasticity of aggregate trade, restricted to trade flows over distances above 2,000 km, also from eq. (6). Column 7 shows the predicted asymptotic distance elasticity of trade, $\zeta_{theory} = 1 + 2(\lambda - 1)/\mu$, with bootstrapped standard errors (999 bootstraps). Columns 8 and 9 show the p-values of χ^2 Wald tests of $\zeta_{all} = \zeta_{theory}$ and $\zeta_{long} = \zeta_{theory}$, respectively, using 999 bootstrapped estimates of λ and μ. All standard errors are in parentheses (bootstrapped in col. 7 only). All estimates are rounded up to the nearest third decimal.

size, parameter μ in condition (ii), remains relatively stable across firm samples, as does the distance elasticity of aggregate trade, both for all and for long distances (above 2,000 km), parameters ζ_{all} and ζ_{long}.

The gradual increase in the Pareto tail index λ is to be expected, given the departure from a Pareto distribution for small and large firms that is visible in figure 1: the counter–cumulative distribution is concave on a log-log scale. This is a well-documented observation (Axtell 2001).

The gradual increase in λ implies that condition (iii) is eventually violated: for export thresholds of 4.52 million French francs and above (i.e., about half or fewer of the original firms), condition (iii) is violated at conventional statistical levels (near or above 5%). When condition (iii) is violated, proposition 1 no longer applies, so it no longer predicts whether the actual distance elasticity of trade (for all distances, ζ_{all}, and distances above 2,000 km, ζ_{long}) ought to be equal to $1 + 2(\lambda - 1)/\mu$ or not. In practice, the last three rows of table 2 show that when condition (iii) is violated at conventional statistical levels (near or above 5%), the difference between the actual distance elasticity of trade (ζ_{all} or ζ_{long}) and $1 + 2(\lambda - 1)/\mu$ is statistically significant at conventional levels (p-values near or below 5%).

IV. Conclusion

I am grateful to Dewitte (2022) for uncovering several coding inconsistencies in Chaney (2018), which I am solely responsible for. This note shows that correcting those inconsistencies leads to statistical tests that are weaker than those in the original version. However, those tests remain statistically significant at conventional levels.

Going beyond a simple replication, I show that the main prediction in Chaney (2018) is robust to sample selection: in subsamples where the conditions identified in Chaney (2018) hold, statistical tests are significant; where those conditions are violated, the tests are insignificant. The reason why the conditions hold for some subsamples and not for others is well known: the distribution of firm sizes departs from a Pareto distribution for small and large firms (Axtell 2001).

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