Network Business Cycles

Thomas Chaney

Toulouse School of Economics

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Preliminary...

Questions

- What drives aggregate fluctuations?
- Are firm level idiosyncratic shocks enough?

Shortcomings of existing theories

1. Macro shocks are elusive:
   - oil/monetary policy shocks, OK
   - preference shocks?
   - technology shocks?
   - mark-up shocks?

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    - preference shocks?
    - technology shocks?
    - mark-up shocks?
2. Large firms/sectors granular hypothesis (Gabaix):
   $\hookrightarrow$ somewhat tautological.
3. Input/output linkages explanation (Acemoglu et al.):
    - input/output linkages generate large sectors.
    - large sectors contribute to aggregate fluctuations.

   $\hookrightarrow$ network structure irrelevant, only size matters (Baqaee 2015)
   $\hookrightarrow$ explanation similar to Gabaix's granular hypothesis.

Ingredients

1. non-convexities (financing frictions).
2. local interactions (input-output linkages).
3. small shocks to small firms.
4. dynamics.

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$$(4. + 3.) + (2. + 1.) \Rightarrow \text{"avalanches" (rare and large)}$$

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   $\hookrightarrow$ if "pyramid" of fragile firms, small shock $\Rightarrow$ large "avalanche".
3. "Small" shocks:
   - Final goods supplier face stochastic demand shocks.
   - No aggregate shocks (e.g. demand high/low for half the firms).

Roadmap

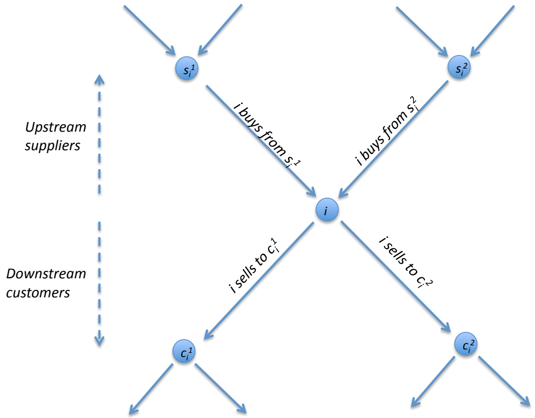
1. Simple (mechanical) theory.
2. Micro-founded theory.
3. Empirical evidence.

1- Simple Theory

Model: set-up (Bak, Chen, Scheinkman, Woodford 1993)

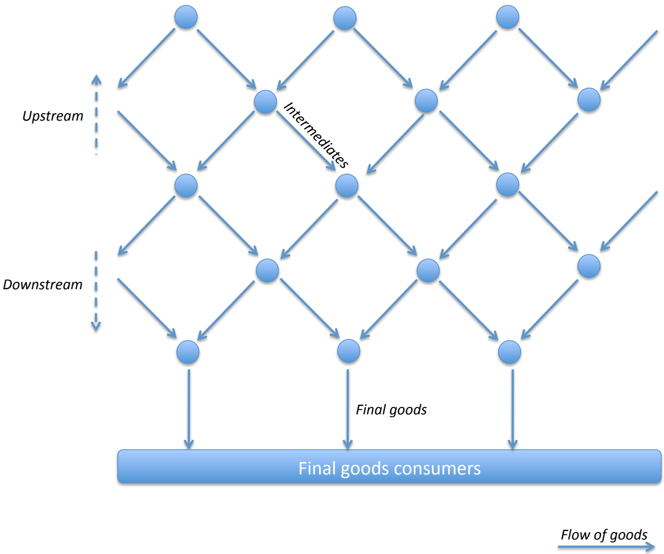
- N^2 firms.
- N production stages / N firms at each stage.
- each firm has:
  - 2 downstream customers
  - 2 upstream suppliers

Input-output linkages



Firm i sells to 2 customers: c_i^1 and c_i^2
Firm i buys from 2 suppliers: s_i^1 and s_i^2

Input-output linkages



Technology

- One exogenous state variable:
  1. demand (low/medium/high)
- One endogenous state variable:
  1. liquidity (low/high)
- Three control variables:
  1. technology (backward/advanced)
  2. output supply (low/high)
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- Technologies:
  backward: low fixed cost, high variable cost
  advanced: high fixed cost, low variable cost

Decision rule

- High liquidity:
    - low demand $\Rightarrow$ backward technology, low output/inputs

  $\hookrightarrow$ liquidity remains high
    - medium demand $\Rightarrow$ advanced technology, high output/inputs

  $\hookrightarrow$ liquidity becomes low
    - high demand $\Rightarrow$ advanced technology, high output/inputs

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- Low liquidity:
    - low demand $\Rightarrow$ backward technology, low output/inputs

  $\hookrightarrow$ liquidity remains low
    - medium demand $\Rightarrow$ backward technology, low output/inputs

  $\hookrightarrow$ liquidity becomes high
    - high demand $\Rightarrow$ advanced technology, high output/inputs

  $\hookrightarrow$ liquidity remains low

“Bad” avalanches (downturn)

- Each period, 1 final good demand is low (all others are high).
  - backward technology for this final goods producer.
  - low demand for both of its input suppliers.
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  - they pass along medium demand for inputs to their suppliers.
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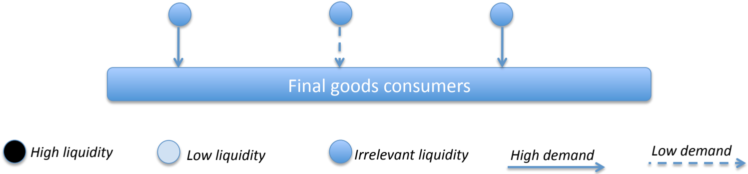
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High liquidity
Low liquidity
Irrelevant liquidity
High demand
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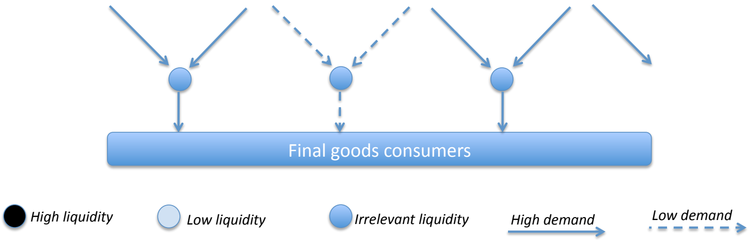
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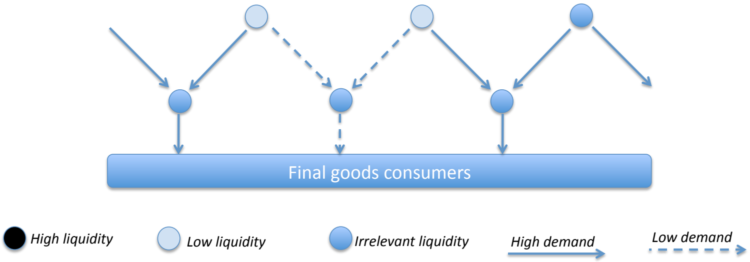
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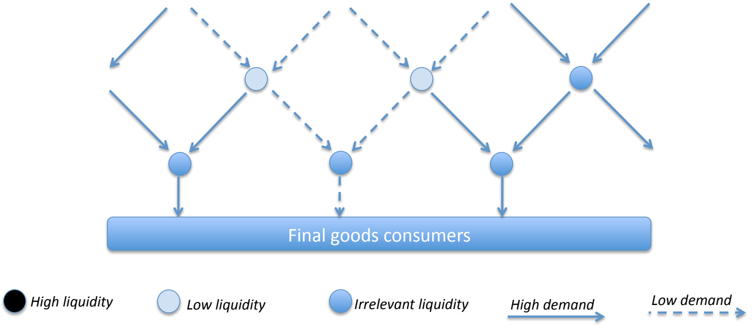

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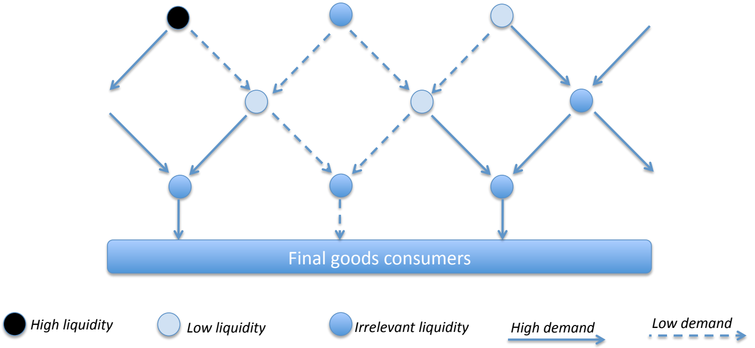
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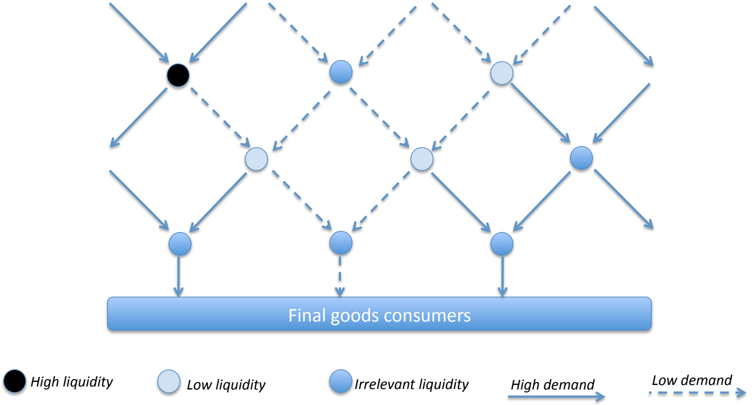

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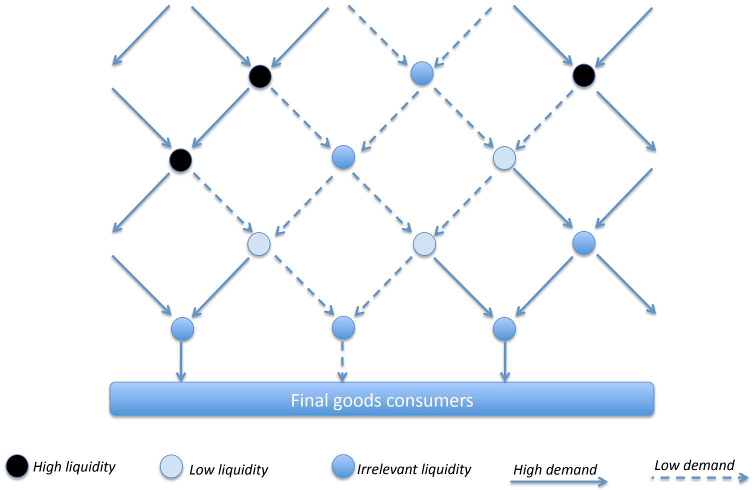

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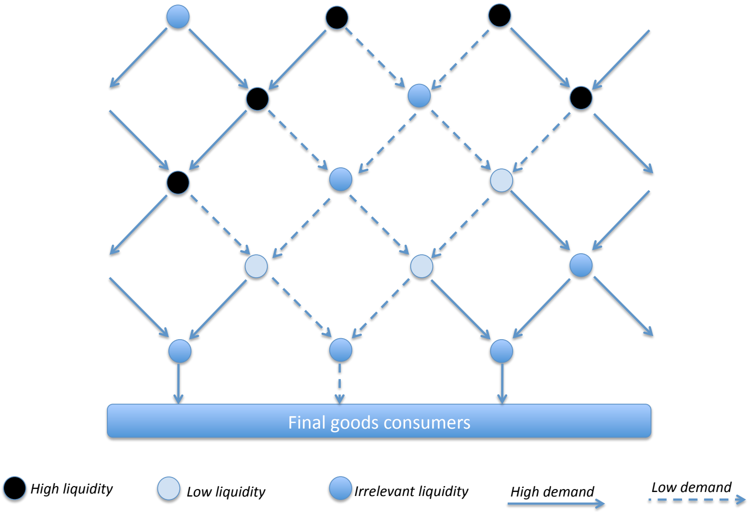

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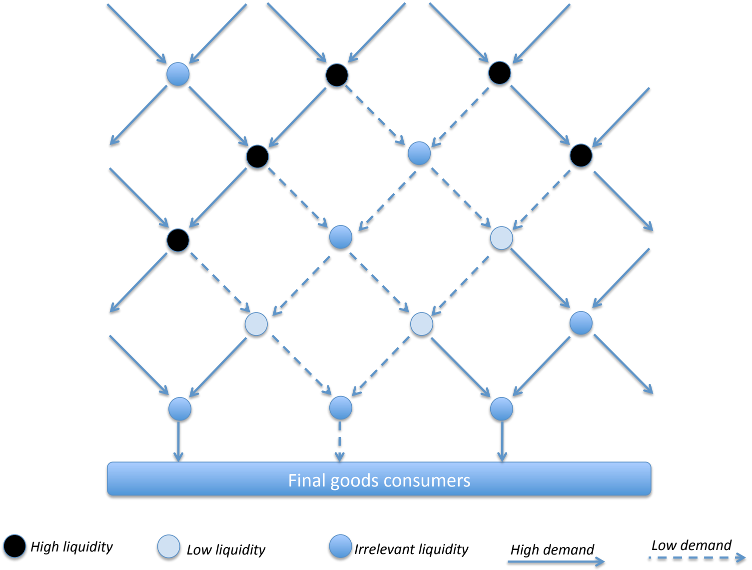
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Aggregate fluctuations

- If one single high final demand at a time (all others are low):

$$\Pr\left(\widetilde{\frac{\Delta Y}{Y}} > \frac{\Delta Y}{Y}\right) \propto \left(\frac{\Delta Y}{Y}\right)^{-1/3}$$

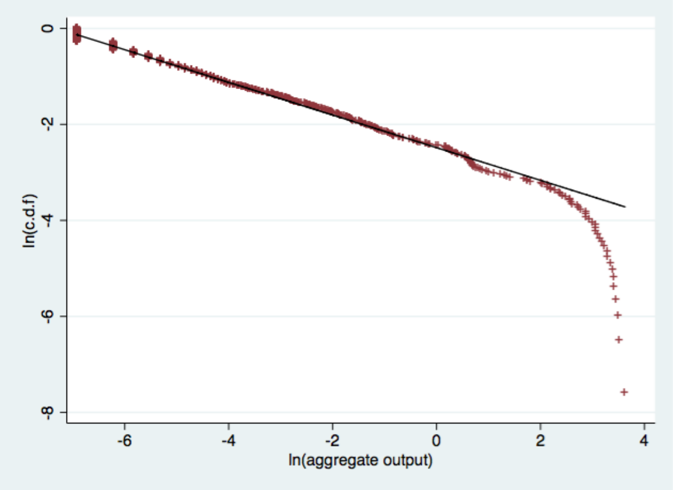
- If one single low final demand at a time (all others are high):

$$\Pr\left(-\widetilde{\frac{\Delta Y}{Y}} < -\frac{\Delta Y}{Y}\right) \propto \left(\frac{\Delta Y}{Y}\right)^{-1/3}$$

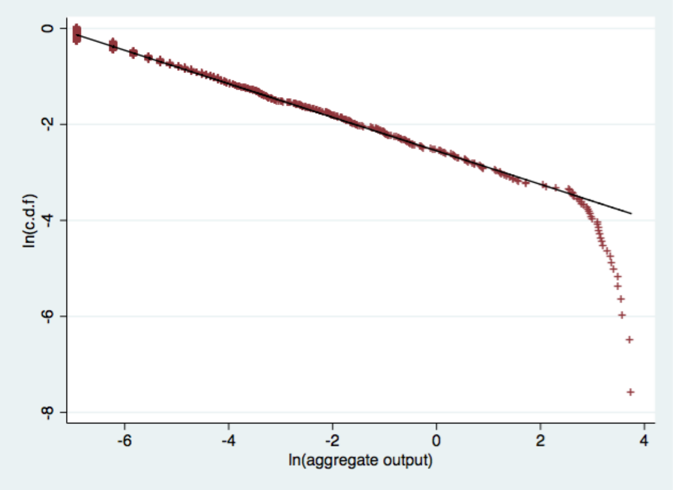
- If half final demand is high, half is low:

$$\frac{\Delta Y}{Y} \sim \mathcal{N}\left(0, \sigma^2\right)$$

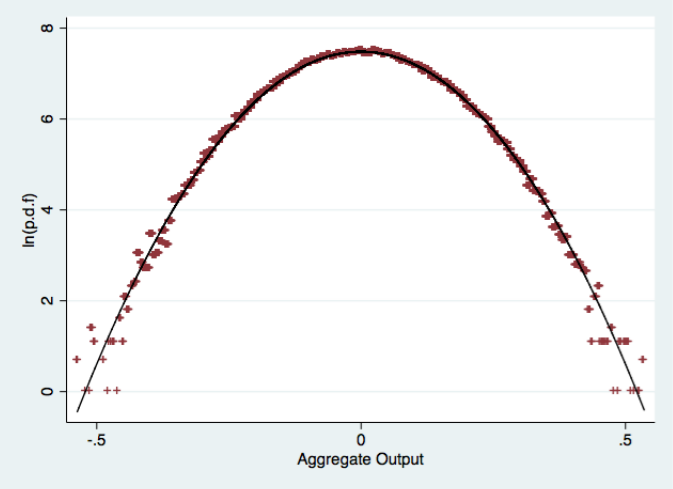
Positive avalanches (one singe high demand)



Negative avalanches (one singe low demand)



Average avalanches (50/50 high-low final demand)



2- Micro-founded Theory

Set-up

- Final goods $\rightarrow$ final consumer.
- Intermediate inputs $\rightarrow$ other firms.
- Two factors of production:
    1. equipped labor
    2. intermediates

Preferences

- One representative agents has CES preferences:

$$U = \left(\sum_{n \in \mathcal{S}_{final}} \left(D_n Q_n \right)^{\frac{\sigma - \mathbf{1}}{\sigma}} \right)^{\frac{\sigma}{\sigma - \mathbf{1}}}$$

$\mathcal{S}_{final}$: set of final goods.
Q_n: consumption of good n.
D_n: demand shocks.
σ: elasticity of substitution.

Technologies

- Firms combine labor and intermediates:

$$Q_n = \left((T_n L_n)^{\frac{\sigma-1}{\sigma}} + \sum_{s \in \mathcal{S}_n} M_s^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

T_n: labor augmenting technology shock.
L_n: labor demand.
$\mathcal{S}_n$: set of suppliers to n.
M_s: demand for intermediate s.
σ: elasticity of substitution.

Sales/revenue

- Firms sell to final goods consumers and/or other firms:

$$R_n = R_{n,final} + R_{n,intermediates}$$

Final good demand

- CES demand function:

$$R_{n,final} = \frac{(p_n / D_n)^{1-\sigma}}{\sum_{s \in \mathcal{S}_{final}} (p_s / D_s)^{1-\sigma}} R_{final}$$

Intermediates demand

- CES demand function:

$$R_{n,intermediates} = (1 - 1/\sigma) \sum_{c \in \mathcal{C}_n} \frac{p_n^{1-\sigma}}{(w/T_c)^{1-\sigma} + \sum_{s \in \mathcal{S}_c} p_s^{1-\sigma}} R_c$$

Pricing

- Simplified Dixit-Stiglitz constant mark-ups:

$$p_n = \frac{\sigma}{\sigma - 1} \left((w / T_n)^{1-\sigma} + \sum_{s \in \mathcal{S}_n} p_s^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$$

Equilibrium step 1: supply (prices)

- Scaled price: $\pi_n = p_n^{1-\sigma}$.
- Scaled inverse mark-up: $\alpha = (1 - 1/\sigma)^{\sigma-1}$.
- Scaled technology: $\tau_n = T_n^{\sigma-1}$.
- Endogenous prices:

$$\pi_n = \alpha \left(\tau_n + \sum_{s \in \mathcal{S}_n} \pi_s \right)$$

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- Simultaneous pricing equations (**M**: input-output matrix):

$$\pi = \alpha \left(\mathbf{I}\tau + \mathbf{M}\pi \right)$$

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- Equilibrium prices:

$$\begin{aligned} \pi &= \alpha \left(\mathbf{I} - \alpha \mathbf{M} \right)^{-1} \tau \\ &= \alpha \left(\sum_{k \geq 0} \left(\alpha \mathbf{M} \right)^k \right) \tau \end{aligned}$$

Equilibrium step 2: supply (final goods price index)

- Ideal price index:

$$P_{final} = \left(\sum_{s \in \mathcal{S}_{final}} \left(\frac{p_s}{D_s} \right)^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$$

Equilibrium step 3: demand (sales/revenues)

- Scaled price: $\pi_n = p_n^{1-\sigma}$.
- Scaled inverse mark-up: $\beta = (1 - 1/\sigma)^{\sigma}$.
- Scaled demand shock: $\delta_n = D_n^{\sigma-1}$.
- Endogenous revenue:

$$(R_n/\pi_n) = \left(R_{final}/P_{final}^{1-\sigma}\right)\delta_n + \beta \sum_{c \in \mathcal{C}_n} (R_c/\pi_c)$$

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- Simultaneous revenue equations ($\mathbf{M}'$: output-input matrix):

$$(R \oslash \pi) = \left(R_{final} / P_{final}^{1-\sigma}\right) \delta + \beta \mathbf{M}' (R \oslash \pi)$$

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$$\left(R_n / \pi_n\right) = \left(R_{final} / P_{final}^{1-\sigma}\right) \delta_n + \beta \sum_{c \in \mathcal{C}_n} \left(R_c / \pi_c\right)$$

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- Equilibrium revenues:

$$R = \left(R_{final} / P_{final}^{1-\sigma}\right) \pi \circledast \left(\sum_{k \geq 0} \left(\beta \mathbf{M}'\right)^k\right) \delta$$

Equilibrium step 4: supply+demand

- Equilibrium sales:

$$R = \alpha \left(R_{final} / P_{final}^{1-\sigma} \right) \left(\sum_{k \geq 0} \left(\alpha \mathbf{M} \right)^k \right) \left(\sum_{k \geq 0} \left(\beta \mathbf{M}' \right)^k \right) \left(\tau \circledast \delta \right)$$

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- **Interpretation.** a firm may be large because:
  1. it is productive (τ_n large).
  2. its products are appealing (δ_n large).
  3. it is an important customer ($\sum_{k \geq 0} (\alpha \mathbf{M})^k$ large).
  4. it is an important supplier ($\sum_{k \geq 0} (\beta \mathbf{M}')^k$ large).

Adding shocks

- Technology shocks:

$$\tilde{\tau} = \tau \circledast \tilde{\epsilon}_\tau$$

- Demand shocks:

$$\tilde{\delta} = \delta \circledast \tilde{\epsilon}_\delta$$

- Shocks are i.i.d.:

$$\tilde{\epsilon}_{\tau,n}, \tilde{\epsilon}_{\delta,n} \sim \mathcal{N}(0,1)$$

Aggregate fluctuations

- Sum of individual shocks:

$$\tilde{R} = \alpha \left(R_{final} / P_{final}^{1-\sigma} \right) \left[\left(\sum_{k \geq 0} (\alpha \mathbf{M})^k \right) \left(\sum_{k \geq 0} (\beta \mathbf{M}')^k \right) (\tau \circledast \delta) \right] \circledast (\tilde{\epsilon}_\tau \circledast \tilde{\epsilon}_\delta)$$

Network irrelevance

- Only size matters (see Gabaix 2011, Baqaee 2015).
- Consider two economies $(\mathbf{M}_1, \tau_1, \delta_1)$ and $(\mathbf{M}_2, \tau_2, \delta_2)$:

$$\left[\left(\sum_{k\geq 0}(\alpha M_1)^k\right)\left(\sum_{k\geq 0}(\beta \mathbf{M}_1')^k\right)(\tau_1 \circledast \delta_1)\right]$$
$$=$$
$$\left[\left(\sum_{k\geq 0}(\alpha M_2)^k\right)\left(\sum_{k\geq 0}(\beta \mathbf{M}_2')^k\right)(\tau_2 \circledast \delta_2)\right]$$

$$\Rightarrow \tilde{R}_1 = \tilde{R}_2$$

Dynamics

- Dynamics with financing frictions (as in Bak et al.).
- Correlation of (endogenous) technology choices.
- Aggregate fluctuations from idiosyncratic shocks with “small” firms.

Conclusion

- Is it possible to generate aggregate fluctuations from small shocks to small firms?
- In a static equilibrium, no.
- In a dynamic equilibrium, yes.

Thank you