7 Appendix Not For Publication

This unpublished appendix to "Exchange Rate Pass-Through in a Competitive Model of Pricing-to-Market" (by Raphael Auer and Thomas Chaney) contains additional proofs and the data description.

7.1 Technical Appendix: proofs

7.1.1 Proof of proposition 3

Proposition 3 (reminded) *There exists a* $(p(\cdot),\ v(\cdot),\ p_{\min},\ q_{\min})$ *solution to Equations (5), (6), (7) and (8), not necessarily unique.*

Let us assume for simplicity that the cost function is quadratic, so that $c^{-1}(p) = p$, and that $\tau w_H = 1$. As pointed out by Rochet and Stole (2002, p. 282, footnote 10),[14] this is not a restrictive assumption. As they argue, the cost function could be any strictly convex function: "since the measurement of units of consumers' [valuations] and product qualities are not intrinsic, they can be redefined in such a way that costs are quadratic [...]".

Before turning to the proof of proposition 3, it will be useful to first prove the following lemma.

Lemma 1 *There exists a unique* α *solution to*

$$\begin{cases} \alpha = \frac{p_{\min}}{F_v^{-1}\left[\frac{N}{L}p_{\min}F_q(\beta(\delta(\alpha)))\right]} \\ \beta(\delta) = \frac{p_{\min}}{F_v^{-1}\left[\frac{N}{L}\delta\right]} \\ \delta(\alpha) = \int_\alpha^{q_{\max}} F_v^{-1}\left[\frac{N}{L}\int_\chi^{q_{\max}} p_{\min} f_q(q)\,dq\right] d\chi + p_{\min} \end{cases}$$

Proof. In the third equation, the function δ is continuously decreasing in α. Since F_v^{-1} is a decreasing function, in the second equation, the denominator is decreasing in α, so that the function β is increasing in $\delta(\alpha)$. In the first equation, the counter-cumulative function F_q is decreasing, while F_v^{-1} is decreasing, so that the denominator is increasing in β. $\beta(\alpha)$ is increasing in α. Consequently, in the first equation, the denominator is increasing in α. Therefore the right-hand side of the first equation continuously decreases in α, crossing the 45° line only once. ■

We can now turn to the proof of the existence of an equilibrium.

Proof. It is straightforward to prove that the zero cutoff profit condition (5) determines a unique price $p_{\min}$. Equation (6) mechanically defines the matching function $v(\cdot)$. We now prove that there exists a solution $(p(\cdot), \ q_{\min})$ to Equations (7) and (8).

Let E be the set of continuous functions from any interval $I \subset [\alpha, \beta]$ to $[\gamma, \delta]$ normed by $\|(p, q_{\min})\| = \sqrt{\sup_q |p(q)|^2 + |q_{\min}|^2}$. α, β, γ and δ are positive real numbers (defined below). Let Γ be a mapping from $S = E \times [\alpha, \beta]$ into itself (proven below), such that $\Gamma(p_1, q_1) = (p_2, q_2)$ is defined as follows:

$$\begin{cases} p_2(q) = \int_{q_1}^{q} \bar{F}_v^{-1}\left[\frac{N}{L}\int_{\xi}^{q_{\max}} p_1(\chi) f_q(\chi)\, d\chi\right] d\xi + p_{\min}, \ \forall Q \in [q_1, q_{\max}] \\ q_2 = \frac{p_{\min}}{F_v^{-1}\left[\frac{N}{L}\int_{q_1}^{q_{\max}} p_1(q) f_q(q) dq\right]} \end{cases}$$

α is defined in Lemma 1. δ is defined by $\delta(\alpha)$ as in Lemma 1. $\gamma = p_{\min}$ and β is defined by $\beta(\delta)$ as in Lemma 1.

- S is a Banach space: the set of continuous functions over a closed interval of the real line, normed by the sup norm, is a Banach space; the Cartesian product of this space and a closed interval with the Euclidean norm is a Banach space, too. Since Cauchy sequences converge in both E with the sup norm, and in $[\alpha, \beta]$ with the absolute value norm, then Cauchy sequences converge in S with the conjugated norm.

- Γ maps S into itself, or, if $(p_1, q_1) \in S$, then $\Gamma(p_1, q_1) = (p_2, q_2) \in S$:

  - if $p_1 \in E$, then by construction, $\bar{F}_v^{-1}$ and f_q being continuous, p_2 is continuous.
  - F_v^{-1} takes only positive values, so for $q \in [q_1, q_{\max}]$, $p_2(q) \geq p_{\min} = \gamma$.
  - F_v^{-1} takes only positive values, so for $q \in [q_1, q_{\max}]$, $p_1(q) \geq p_{\min}$. F_v^{-1} is decreasing and takes only non-negative values so that $p_2(q) \leq \int_{\alpha}^{q_{\max}} \bar{F}_v^{-1}\left[\frac{N}{L}\int_{\xi}^{q_{\max}} p_{\min} f_q(\chi)\, d\chi\right] d\xi + p_{\min} = \delta$.
  - for any $q \in [q_1, q_{\max}]$, $p_1(q) \geq p_{\min}$. Therefore, for $q_1 \in [\alpha, \beta]$, $\int_{q_1}^{q_{\max}} p_1(\chi) f_q(\chi)\, d\chi \geq p_{\min} F_q(\beta)$. F_v^{-1} is decreasing, so that $q_2 \geq \alpha$.

- for any $q \in [q_1, q_{\max}]$, $p_1(q) \leq \delta$. Moreover, f_q is a well-defined density function, so that $\int_{q_1}^{q_{\max}} p_1(\chi) f_q(\chi) d\chi \leq \frac{N}{L}\delta$. F_v^{-1} is decreasing so that $q_2 \leq \beta$.
- We have therefore proven that if $(p_1, q_1) \in S$, then $\Gamma(p_1, q_1) = (p_2, q_2) \in S$: $p_2 \in E$ (it is a continuous function that is from an interval included in $[\alpha, \beta]$ into $[\gamma, \delta]$), and $q_2 \in [\gamma, \delta]$.

- Γ is continuous, or $\forall\, \varepsilon > 0,\ \exists\, \delta > 0$ such that if $\|(p_1, q_1) - (p_1', q_1')\| \leq \delta$, then $\|\Gamma(p_1, q_1) - \Gamma(p_1', q_1')\| \leq \varepsilon$, for any (p_1, q_1) and (p_1', q_1') in S.

- Applying the Schauder fixed point theorem, there exists a fixed point (not necessarily unique) $(p, q_{\min})$ such that $(p, q_{\min}) = \Gamma(p, q_{\min})$

■

7.1.2 Proof of proposition 4

Proposition 4 (reminded) *If the entry cost is such that* $f^E = \left(\frac{\lambda_q - \eta}{(1+\eta)(\lambda_q + \lambda_v)}\right)^{1+\eta}$, *then there exists a unique equilibrium price schedule, lowest quality exported, and lowest price, defined as*

$$\begin{cases} p(q) = \gamma (\tau w_H)^{\eta/(\lambda_v+\eta)} q^{(\lambda_v+\lambda_q)/(\lambda_v+\eta)} + \tau w_H \\ q_{\min} = \gamma' (\tau w_H)^{\lambda_v/(\lambda_v+\lambda_q)} \\ p_{\min} = \left(\frac{\lambda_q+\lambda_v}{\lambda_q-\eta}\right) \tau w_H \end{cases}$$

where γ and γ' are constants.[15]

Proof. An equilibrium is defined by the following 4 equations:

$$\begin{cases} v(q) = \bar{F}_v^{-1}\left(\frac{N_H}{L_F}\int_q^{\infty} c^{-1}\left(\frac{p(x)}{\tau w_H}\right) f_q(x)\, dx\right) \\ p(q) = \int_{q_{\min}}^{q} v(\chi)\, d\chi + p_{\min} \\ p_{\min} = v(q_{\min})\, q_{\min} \\ \tau w_H f^E = p_{\min} S(p_{\min}) - \int_0^{S(p_{\min})} \tau w_H c(s)\, ds \end{cases}$$

where $\bar{F}_v$ is the "counter cumulative distribution" of the valuations v. In our closed-form example, we have the following functional forms,

$$\begin{cases} \bar{F}_v^{-1}(m) = \bar{v} m^{-1/\lambda_v} \\ f_q(q) = \lambda_q \left(\frac{q}{\bar{q}}\right)^{-\lambda_q} q^{-1} \\ c^{-1}\left(\frac{p}{\tau w_H}\right) = \left(\frac{p}{\tau w_H} - 1\right)^{\eta} \end{cases}$$

We guess that the equilibrium price schedule is of the following form

$$p(q) = \alpha \tau w_H q^{\beta} + \tau w_H$$

with α and β being some positive constant to be determined. We have 5 equations and 6 unknowns $(v(\cdot),\ p(\cdot),\ p_{\min},\ q_{\min},\ \alpha,\ \beta)$. The condition guaranteeing a unique equilibrium is the size of the fixed entry cost f^E.

Plugging the equilibrium conditions into our guess for the price schedule, the following simple algebra yields

$$\begin{aligned} p(q) &= \int_{q_{\min}}^{q} v(\chi)\, d\chi + p_{\min} \\ &= \int_{q_{\min}}^{q} \bar{F}_v^{-1}\left(\frac{N_H}{L_F}\int_{\chi}^{\infty} c^{-1}\left(\frac{p(x)}{w_H \tau}\right) f_q(x)\, dx\right) d\chi + p_{\min} \\ &= \int_{q_{\min}}^{q} \bar{F}_v^{-1}\left(\frac{N_H}{L_F}\int_{\chi}^{\infty} \left(\frac{p(x)}{\tau w_H} - 1\right)^{\eta} f_q(x)\, dx\right) d\chi + p_{\min} \\ &= \int_{q_{\min}}^{q} \bar{F}_v^{-1}\left(\frac{N_H}{L_F}\int_{\chi}^{\infty} \alpha^{\eta} x^{\beta\eta} \lambda_q \left(\frac{x}{\bar{q}}\right)^{-\lambda_q} x^{-1} dx\right) d\chi + p_{\min} \\ &= \int_{q_{\min}}^{q} \bar{F}_v^{-1}\left(\frac{N_H}{L_F}\frac{\alpha^{\eta}\lambda_q}{\lambda_q - \eta\beta}\chi^{\eta\beta - \lambda_q}\right) d\chi + p_{\min} \\ &= \frac{\bar{v}}{\bar{q}^{\lambda_q/\lambda_v}}\left(\frac{N_H}{L_F}\frac{\alpha^{\eta}\lambda_q}{\lambda_q - \eta\beta}\right)^{-1/\lambda_v} \int_{q_{\min}}^{q} \chi^{(\lambda_q - \eta\beta)/\lambda_v} d\chi + p_{\min} \\ &= \frac{\bar{v}}{\bar{q}^{\lambda_q/\lambda_v}}\left(\frac{N_H}{L_F}\frac{\alpha^{\eta}\lambda_q}{\lambda_q - \eta\beta}\right)^{-1/\lambda_v} \frac{\lambda_v}{\lambda_v + \lambda_q - \eta\beta}\left(q^{(\lambda_q + \lambda_v - \eta\beta)/\lambda_v} - q_{\min}^{(\lambda_q + \lambda_v - \eta\beta)/\lambda_v}\right) + p_{\min} \end{aligned}$$

For our guess to be correct for any quality, it must be the case that

$$\begin{aligned}\beta &= \frac{\lambda_v+\lambda_q}{\lambda_v+\eta}\\ \tau w_H \alpha &= \left(\frac{\lambda_v \bar{v}^{\lambda_v}}{\lambda_q \bar{q}^{\lambda_q}}\left(\frac{\lambda_v+\eta}{\lambda_v+\lambda_q}\right)^{\lambda_v}\frac{\lambda_q-\eta}{\lambda_q+\lambda_v}\frac{L}{N}\right)^{1/(\lambda_v+\eta)} \times (\tau w_H)^{\eta/(\lambda_v+\eta)}\end{aligned}$$

This gives us the following expression for the equilibrium price schedule:

$$\begin{aligned}p(q) &= \gamma (\tau w_H)^{\eta/(\lambda_v+\eta)}\, q^{(\lambda_v+\lambda_q)/(\lambda_v+\eta)} + \tau w_H\\ \text{with } \gamma &= \left(\frac{\lambda_v \bar{v}^{\lambda_v}}{\lambda_q \bar{q}^{\lambda_q}}\left(\frac{\lambda_v+\eta}{\lambda_v+\lambda_q}\right)^{\lambda_v}\frac{\lambda_q-\eta}{\lambda_q+\lambda_v}\frac{L_F}{N_H}\right)^{1/(\lambda_v+\eta)}\end{aligned}$$

Note that $\frac{\lambda_v+\lambda_q}{\lambda_v+\eta} > 1$ iif $\lambda_q > \eta$. We need the assumption that $\lambda_q > \eta$, otherwise, there are too many large firms (λ_q small), or large firms are too big (η large), and the integrals would not converge.

If this equilibrium price schedule holds for every quality, it holds for the lowest quality $q_{\min}$ so that

$$p_{\min} = \gamma (\tau w_H)^{\eta/(\lambda_v+\eta)}\, q_{\min}^{(\lambda_v+\lambda_q)/(\lambda_v+)} + \tau w_H$$

This, together with the equation defining the lowest valuation $q_{\min}$, yields a solution for the lowest price and for the lowest valuation

$$\begin{cases} p_{\min} = \left(\frac{\lambda_q+\lambda_v}{\lambda_q-\eta}\right)\tau w_H\\ q_{\min} = \gamma' (\tau w_H)^{\lambda_v/(\lambda_v+\lambda_q)}\\ \text{with } \gamma' = \left(\frac{\lambda_v+\eta}{\lambda_q-\eta}\right)^{(\lambda_v+\eta)/(\lambda_v+\lambda_q)}\end{cases}$$

However, $p_{\min}$ is independently defined by the zero profit cutoff condition

$$\begin{aligned}\tau w_H f^E &= p_{\min} S(p_{\min}) - \int_0^{S(p_{\min})} \tau w_H c(s)\, ds\\ p_{\min} &= \left(1 + \left((1+\eta) f^E\right)^{1/(1+\eta)}\right)\tau w_H\end{aligned}$$

For our guess to be correct, we need that

$$f^E = \left(\frac{\lambda_q - \eta}{(1+\eta)(\lambda_q + \lambda_v)}\right)^{1+\eta}$$

■

7.1.3 Proof of proposition 5 (exchange rate pass-through)

Proposition 5 (reminded) *There is incomplete pass-through of exchange rate shocks into the price of individual goods. The lower the quality of a good, the higher the pass-through.*

Proof. Recall the definition of $\sigma_{p(q)}$, the elasticity of the price $p(q)$ of a quality q good with respect to the exchange rate,

$$\sigma_{p(q)} \equiv \frac{\partial \ln p(q)}{\partial \ln \tau w_H}$$

From the definition of the equilibrium price schedule in proposition 4,

$$p(q) = \gamma (\tau w_H)^{\eta/(\lambda_v+\eta)} q^{(\lambda_v+\lambda_q)/(\lambda_v+\eta)} + \tau w_H$$

Differentiating with respect to τw_H solves to

$$\begin{aligned} \sigma_{p(q)} &= \frac{\partial \ln p(q)}{\partial \ln \tau w_H} \\ &= 1 - \frac{\lambda_v}{\lambda_v - \eta} \times \frac{1}{1 + \gamma^{-1} (\tau w_H)^{\lambda_v/(\lambda_v+\eta)} q^{-(\lambda_v+\lambda_q)/(\lambda_v+\eta)}} \end{aligned}$$

From this expression, it is straightforward to prove that

$$\begin{aligned} \frac{\partial \sigma_{p(q)}}{\partial q} &< 0 \\ \lim_{q \to +\infty} \sigma_{p(q)} &= \frac{\eta}{\lambda_v + \eta} \\ \lim_{q \to 0} \sigma_{p(q)} &= 1 \end{aligned}$$

Since the lowest quality is strictly above 0, we know that for any $q \geq q_{\min}$, we have,

$$\frac{\eta}{\lambda_v + \eta} < \sigma_{p(q)} < 1$$

There is incomplete pass-through of exchange rate shocks into the prices of individual goods (the elasticity $\sigma_{p(q)}$ is smaller than 1 for all goods), and the lower the quality of a good, the higher the pass-through (the elasticity $\sigma_{p(q)}$ is increasing with the quality q). ■

7.2 Data Description Appendix

US imports - C.I.D. at UC Davis:

Unit value is calculated as the total value of export, including freight and insurance cost, excluding duty, divided by quantity. Observations are expressed in log change, year over year. All variables are winsorized.

Exchange rates - IMF, International Financial Statistics:

Average nominal exchange rates, USD per foreign currency. For countries adopting the Euro, all exchange rates are expressed in USD per 1 Euros also for years before the fixed parity was established, to insure comparability over time. The conversion has been made at the parity established in 1999 or 2001 (for Greece) (See http://www.ecb.int/bc/intro/html/index.en.html#fix). Data come from International Financial Statistics, IMF). Observations are expressed in log change, year over year. All variable are winsorized.

Real Unit Labor costs – OECD:

This reports the annual labor income share calculated for this database as total labor costs divided by the nominal output. The OECD documentation states that: "The term labour income share [. . .] relates to compensation of employees adjusted for the self employed and thus essentially relates to labour income. The division of total labour costs by nominal output is sometimes also referred to as a real unit labour cost - as it is equivalent to a deflated unit labour cost where the deflator used is the GDP implicit price deflator for the economic activity (i.e. sector) concerned". Observations are expressed in log change, year over year. All variables are winsorized.

Consumer Price Index, All items – OECD:

Observations are expressed in log change, year over year. Variables are winsorized.

US gdp growth – OECD:

Observations are expressed in log change, year over year. Variables are winsorized.